



ROYAL INSTITUTE  
OF TECHNOLOGY

# Semiconductor Devices

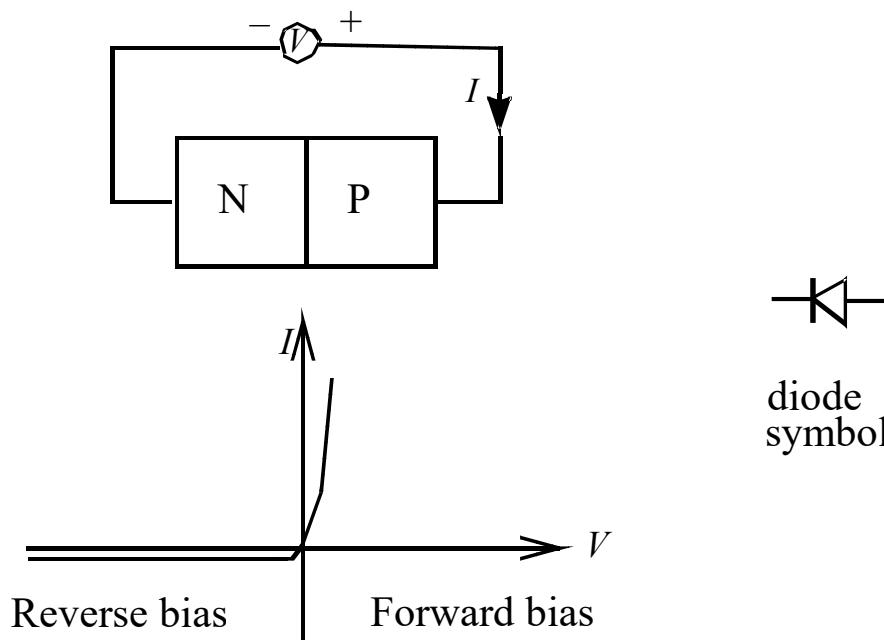
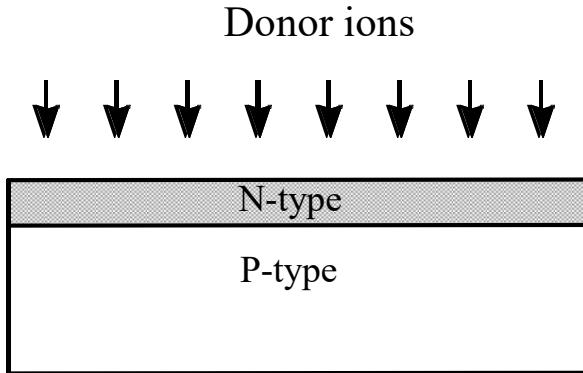
## Spring 2019

Lecture 4

# This Lecture

- Reading
  - 4.1-4.5 PN and Metal Semiconductor Junctions
- Concepts:
  - Joining different materials together (p & n or semiconductor and metal)
  - Two terminal devices, diode, solar-cell, LED (laser), schottky diode, protection circuits (LV), blocking (HV)
  - Some physics including neutral regions, depletion, poisson's equation

## 4.1 Building Blocks of the PN Junction Theory



diode  
symbol

### 4.1.1 Energy Band Diagram of a PN Junction

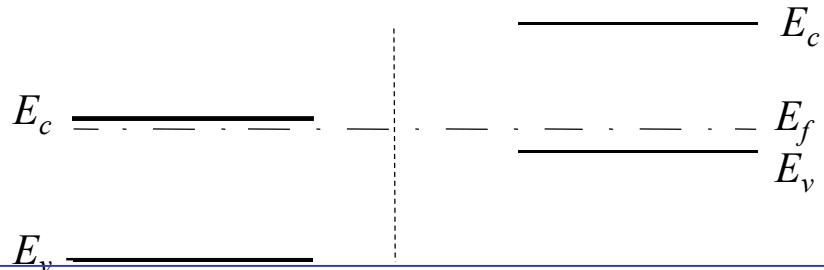
N-region  $\longleftrightarrow$  P-region

(a)



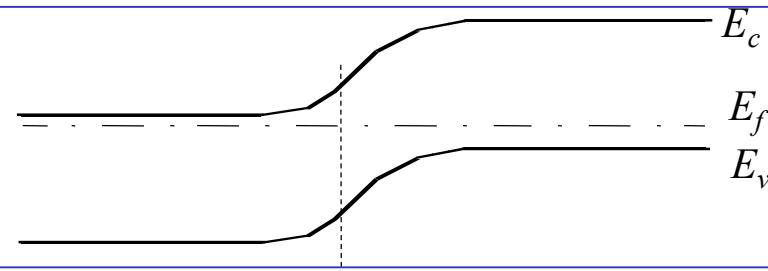
$E_f$  is constant at equilibrium

(b)



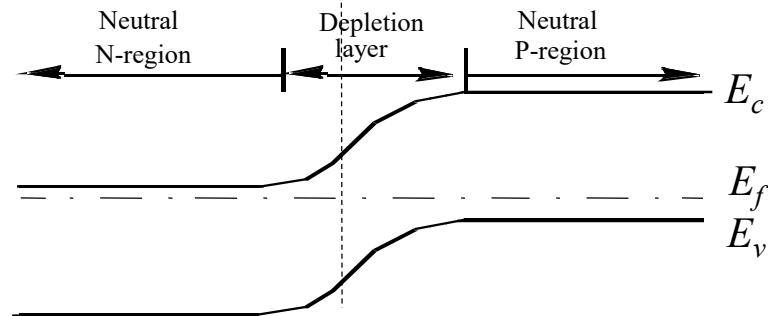
$E_c$  and  $E_v$  are known relative to  $E_f$

(c)



$E_c$  and  $E_v$  are smooth, the exact shape to be determined.

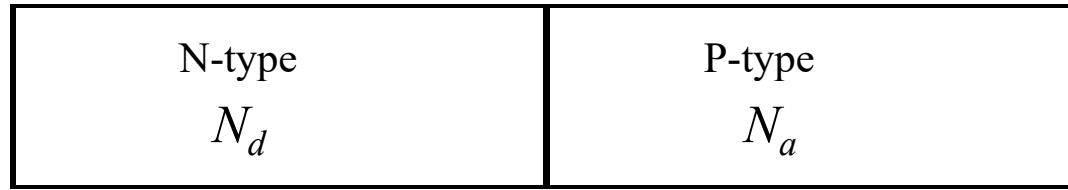
(d)



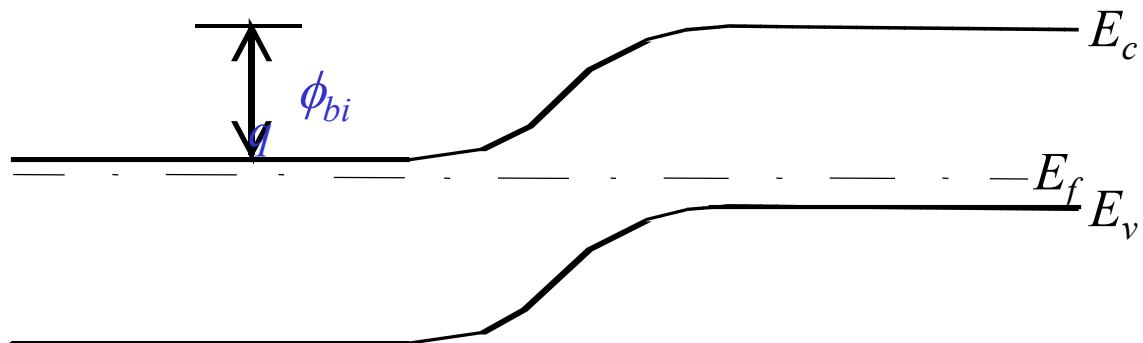
A depletion layer exists at the PN junction where  $n \approx 0$  and  $p \approx 0$ .

## 4.1.2 Built-in Potential

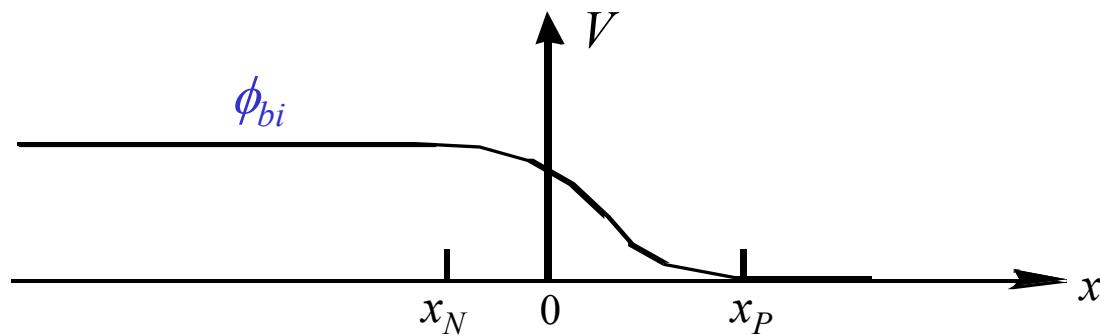
(a)



(b)



(c)



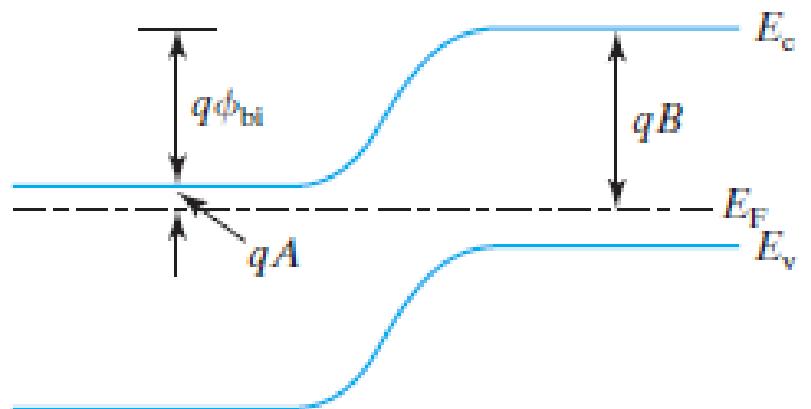
## 4.1.2 Built-in Potential

**N-region**  $n = N_d = N_c e^{-qA/kT} \Rightarrow A = \frac{kT}{q} \ln \frac{N_c}{N_d}$

**P-region**  $n = \frac{n_i^2}{N_a} = N_c e^{-qB/kT} \Rightarrow B = \frac{kT}{q} \ln \frac{N_c N_a}{n_i^2}$

$$\phi_{bi} = B - A = \frac{kT}{q} \left( \ln \frac{N_c N_a}{n_i^2} - \ln \frac{N_c}{N_d} \right)$$

$$\phi_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2}$$



### 4.1.3 Poisson's Equation

Gauss's Law:

$$\varepsilon_s \mathcal{E}(x + \Delta x)A - \varepsilon_s \mathcal{E}(x)A = \rho \Delta x A$$

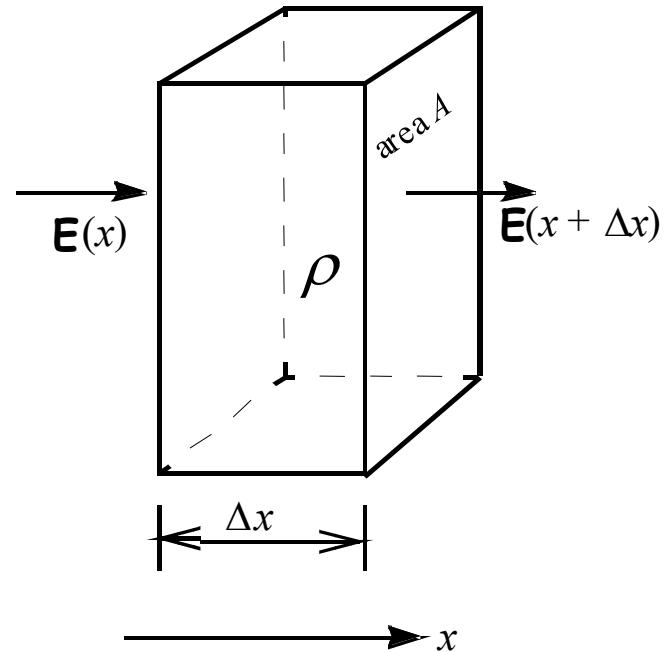
$\varepsilon_s$ : permittivity ( $\sim 12 \varepsilon_0$  for Si)

$\rho$ : charge density ( $\text{C}/\text{cm}^3$ )

$$\frac{\mathcal{E}(x + \Delta x) - \mathcal{E}(x)}{\Delta x} = \frac{\rho}{\varepsilon_s}$$

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon_s}$$

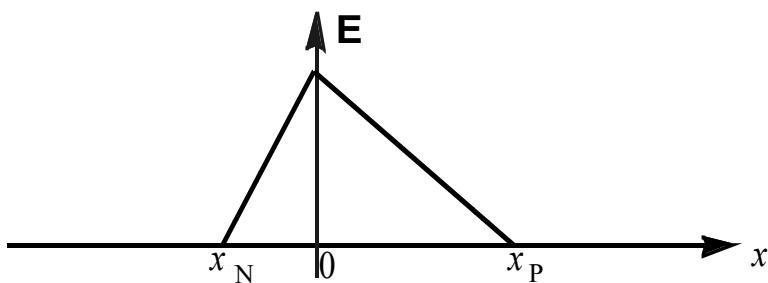
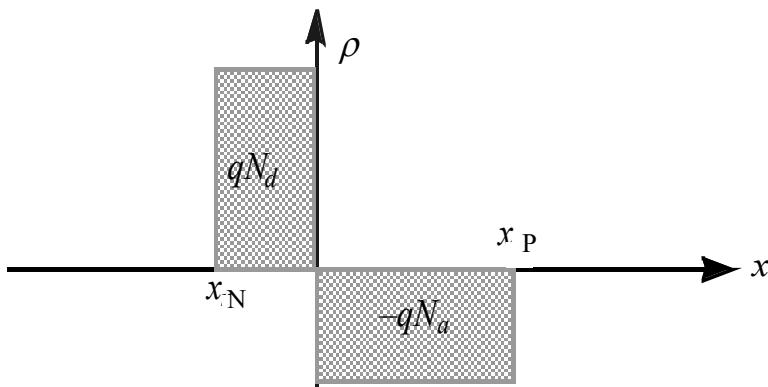
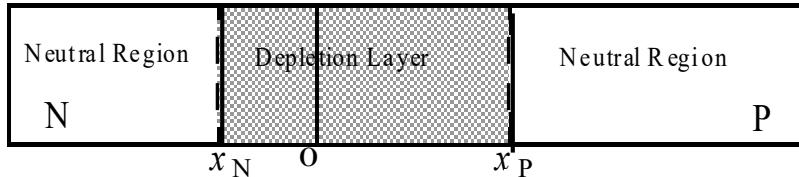
$$\frac{d^2V}{dx^2} = -\frac{d\mathcal{E}}{dx} = -\frac{\rho}{\varepsilon_s}$$



*Poisson's equation*

## 4.2 Depletion-Layer Model

### 4.2.1 Field and Potential in the Depletion Layer



On the *P-side* of the depletion layer,  $\rho = -qN_a$

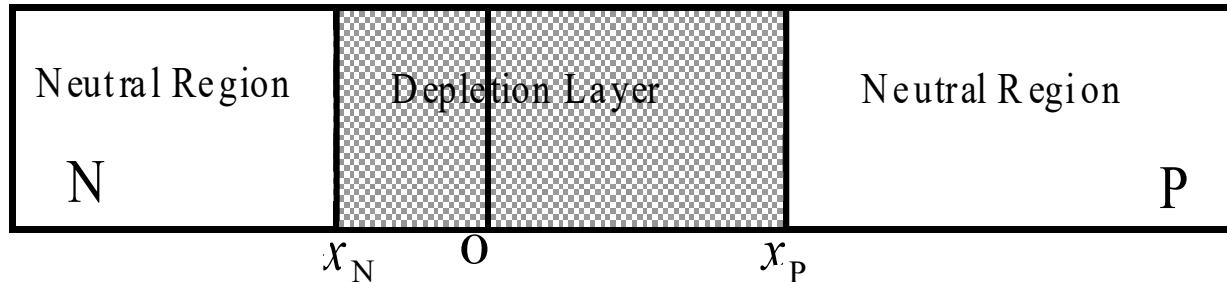
$$\frac{d\mathbf{E}}{dx} = -\frac{qN_a}{\epsilon_s}$$

$$\mathbf{E}(x) = -\frac{qN_a}{\epsilon_s} x + C_1 = \boxed{\frac{qN_a}{\epsilon_s} (x_p - x)}$$

On the *N-side*,  $\rho = qN_d$

$$\mathbf{E}(x) = \frac{qN_d}{\epsilon_s} (x - x_N)$$

## 4.2.1 Field and Potential in the Depletion Layer



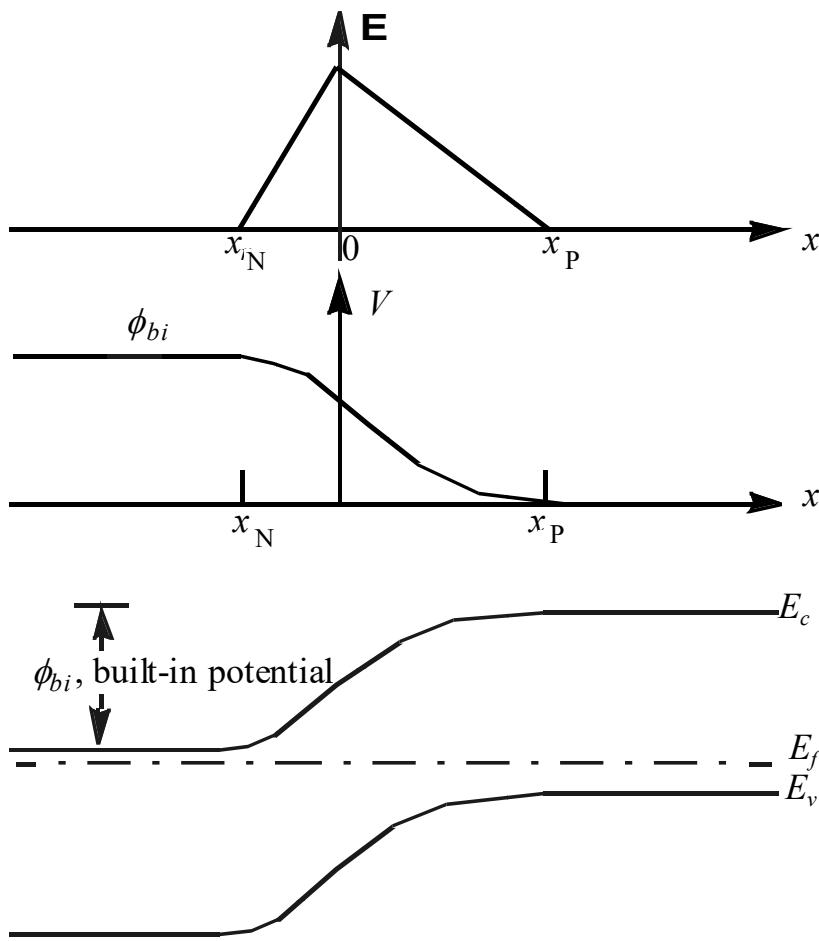
The electric field is continuous at  $x = 0$ .

$$N_a |x_P| = N_d |x_P|$$

Which side of the junction is depleted more?

A one-sided junction is called a  **$N^+P$  junction** or  **$P^+N$  junction**

## 4.2.1 Field and Potential in the Depletion Layer



On the P-side,

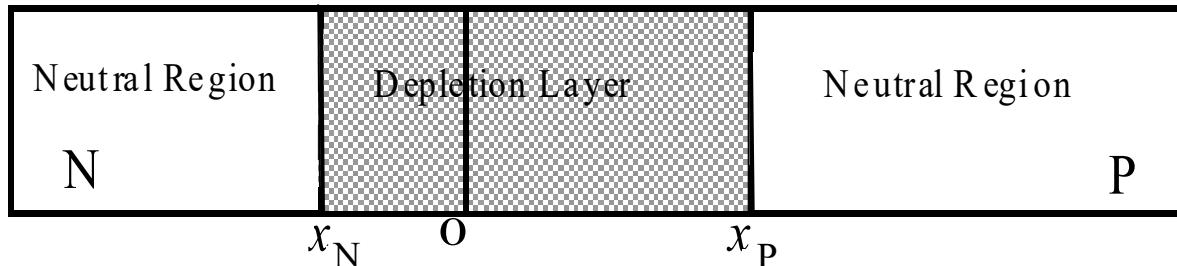
$$V(x) = \frac{qN_a}{2\epsilon_s} (x_p - x)^2$$

Arbitrarily choose the voltage at  $x = x_P$  as  $V = 0$ .

On the N-side,

$$\begin{aligned} V(x) &= D - \frac{qN_d}{2\epsilon_s} (x - x_N)^2 \\ &= \phi_{bi} - \frac{qN_d}{2\epsilon_s} (x - x_N)^2 \end{aligned}$$

## 4.2.2 Depletion-Layer Width



$V$  is continuous at  $x = 0 \rightarrow$

$$x_P - x_N = W_{dep} = \sqrt{\frac{2\epsilon_s \phi_{bi}}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)}$$

If  $N_a \gg N_d$ , as in a P+N junction,

$$W_{dep} = \sqrt{\frac{2\epsilon_s \phi_{bi}}{qN_d}} \approx |x_N|$$

$$|x_P| = |x_N| N_d / N_a \cong 0$$

What about a N+P junction?

$$W_{dep} = \sqrt{2\epsilon_s \phi_{bi} / qN} \quad \text{where} \quad \frac{1}{N} = \frac{1}{N_d} + \frac{1}{N_a} \approx \frac{1}{lighter \; dopant \; density}$$

**EXAMPLE:** A  $P^+N$  junction has  $N_a = 10^{20} \text{ cm}^{-3}$  and  $N_d = 10^{17} \text{ cm}^{-3}$ . What is a) its built in potential, b)  $W_{dep}$ , c)  $x_N$ , and d)  $x_P$ ?

**Solution:**

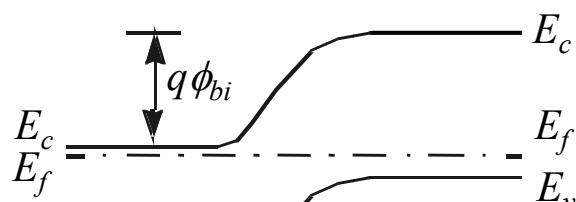
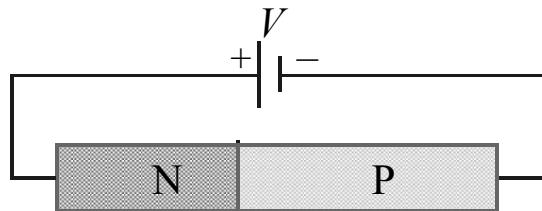
$$a) \phi_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2} = 0.026 \text{ V} \ln \frac{10^{20} \times 10^{17} \text{ cm}^{-6}}{10^{20} \text{ cm}^{-6}} \approx 1 \text{ V}$$

$$b) W_{dep} \approx \sqrt{\frac{2\epsilon_s \phi_{bi}}{qN_d}} = \left( \frac{2 \times 12 \times 8.85 \times 10^{-14} \times 1}{1.6 \times 10^{-19} \times 10^{17}} \right)^{1/2} = 0.12 \mu\text{m}$$

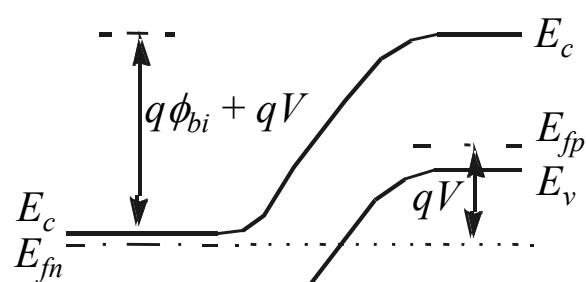
$$c) |x_N| \approx W_{dep} = 0.12 \mu\text{m}$$

$$d) |x_P| = |x_N| N_d / N_a = 1.2 \times 10^{-4} \mu\text{m} = 1.2 \text{ \AA} \approx 0$$

## 4.3 Reverse-Biased PN Junction



(a)  $V = 0$

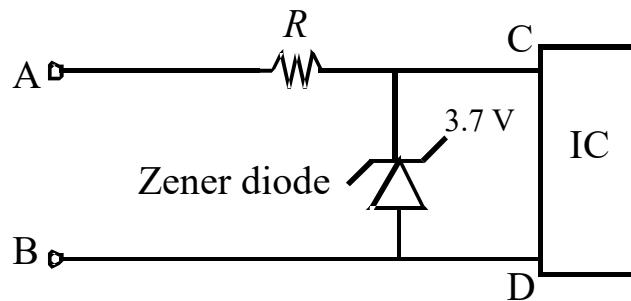
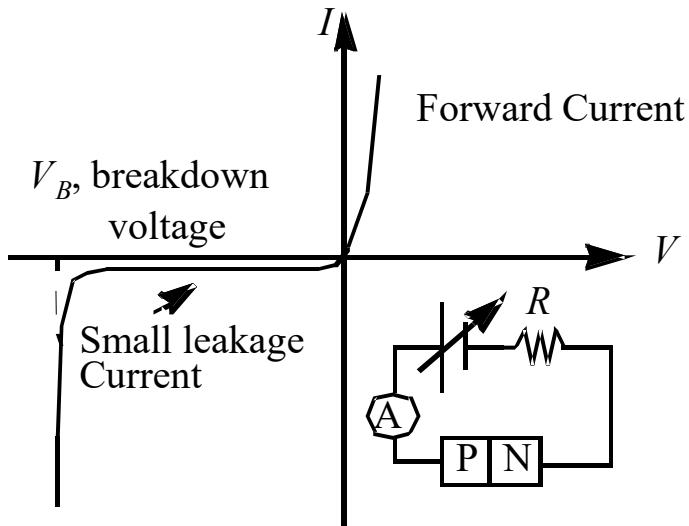


(b) reverse-biased

$$W_{dep} = \sqrt{\frac{2\epsilon_s(\phi_{bi} + |V_r|)}{qN}} = \sqrt{\frac{2\epsilon_s \cdot \text{potential barrier}}{qN}}$$

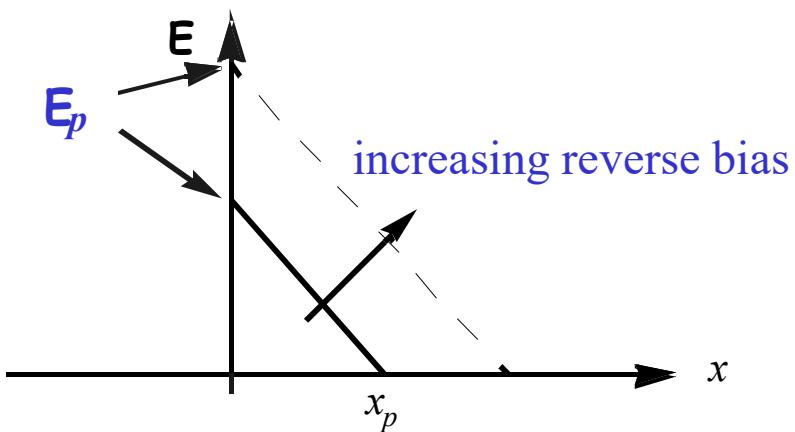
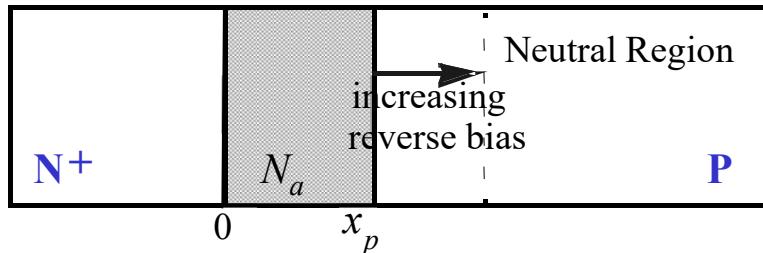
$$\frac{1}{N} = \frac{1}{N_d} + \frac{1}{N_a} \approx \frac{1}{\text{lighter dopant density}}$$

## 4.5 Junction Breakdown



A **Zener diode** is designed to operate in the breakdown mode.

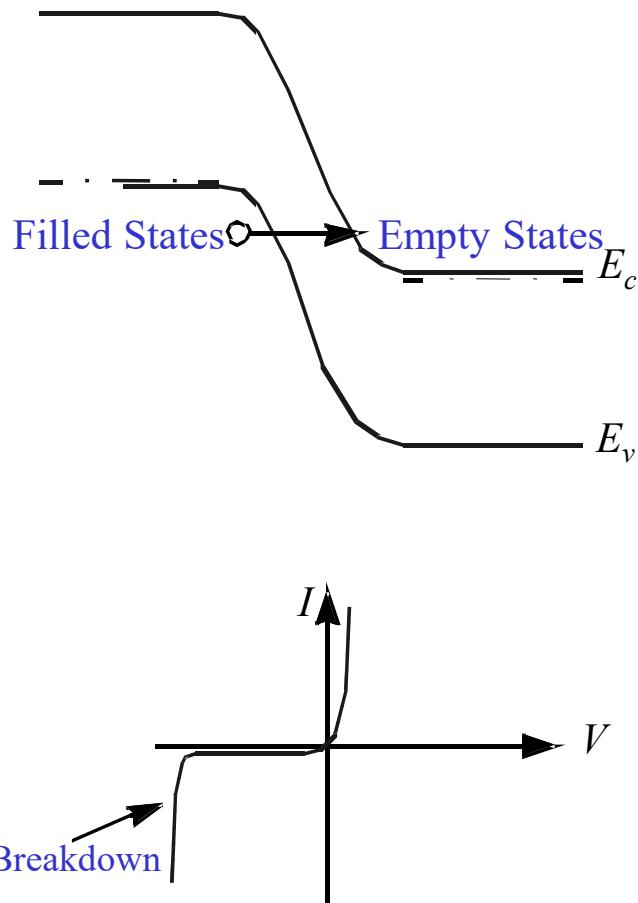
## 4.5.1 Peak Electric Field



$$E_p = E(0) = \left[ \frac{2qN}{\epsilon_s} (\phi_{bi} + |V_r|) \right]^{1/2}$$

$$V_B = \frac{\epsilon_s E_{crit}^2}{2qN} - \phi_{bi}$$

## 4.5.2 Tunneling Breakdown

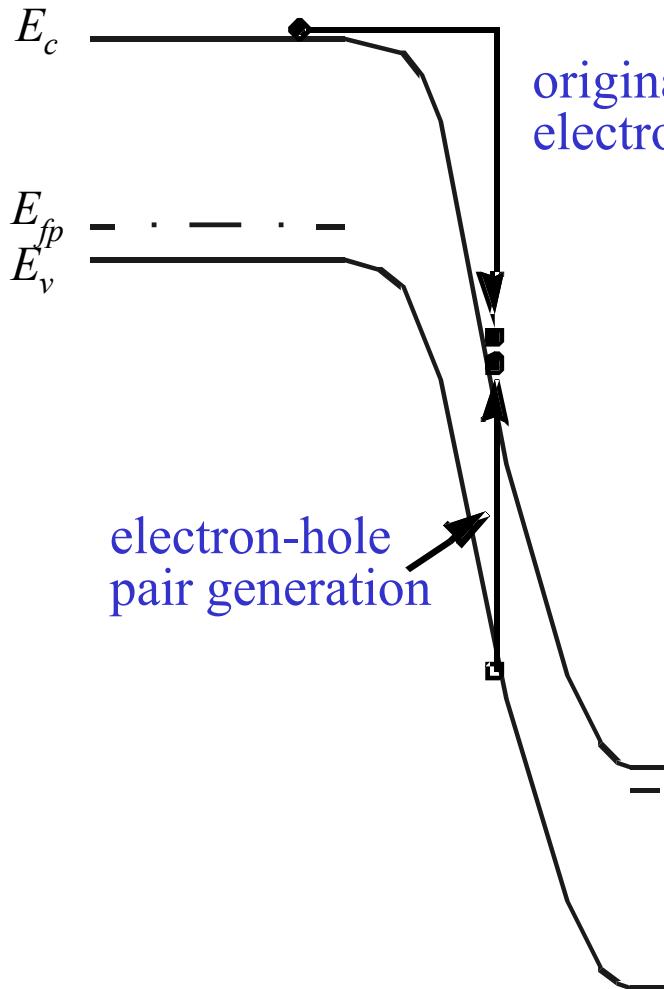


Dominant if both sides of a junction are very heavily doped.

$$J = G e^{-H/\epsilon_p}$$

$$\epsilon_p = \epsilon_{crit} \approx 10^6 \text{ V/cm}$$

### 4.5.3 Avalanche Breakdown

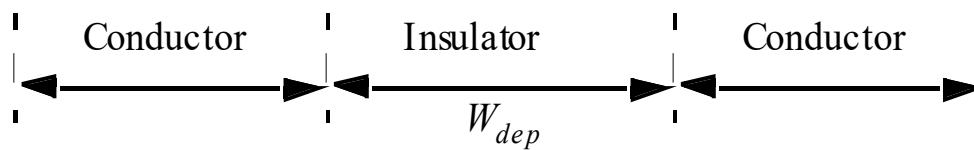
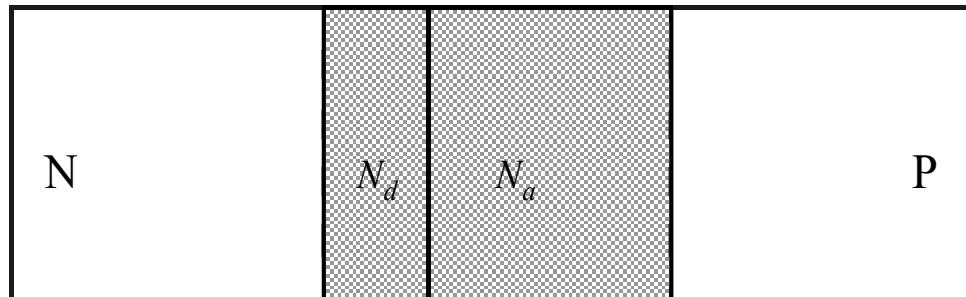


- *impact ionization*: an energetic electron generating electron and hole, which can also cause impact ionization.
- Impact ionization + positive feedback → *avalanche breakdown*

$$V_B = \frac{\varepsilon_s \mathbf{E}_{crit}^2}{2qN}$$

$$V_B \propto \frac{1}{N} = \frac{1}{N_a} + \frac{1}{N_d}$$

## 4.4 Capacitance-Voltage Characteristics

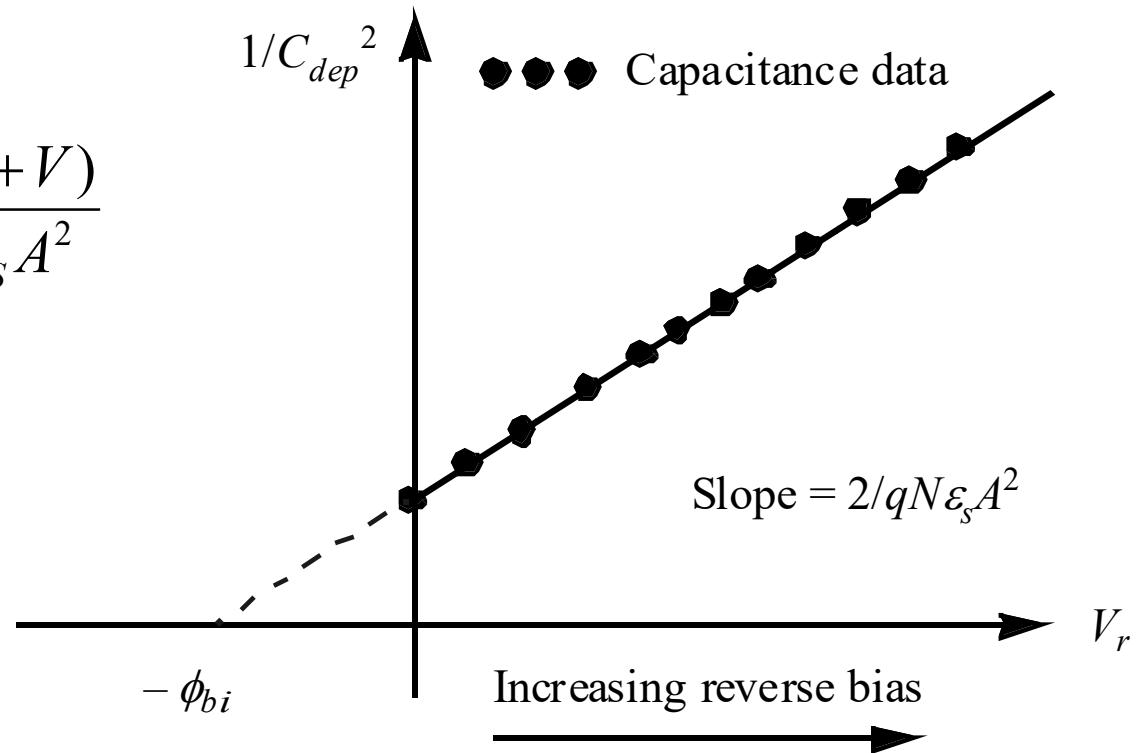


Reverse biased PN junction is  
a capacitor.

$$C_{dep} = A \frac{\epsilon_s}{W_{dep}}$$

## 4.4 Capacitance-Voltage Characteristics

$$\frac{1}{C_{dep}^2} = \frac{W_{dep}^2}{A^2 \epsilon_s^2} = \frac{2(\phi_{bi} + V)}{qN\epsilon_s A^2}$$



- From this C-V data can  $N_a$  and  $N_d$  be determined?

**EXAMPLE:** If the slope of the line in the previous slide is  $2 \times 10^{23} F^{-2} V^{-1}$ , the intercept is  $0.84V$ , and  $A$  is  $1 \mu m^2$ , find the lighter and heavier doping concentrations  $N_l$  and  $N_h$ .

**Solution:**

$$\begin{aligned} N_l &= 2 / (\text{slope} \times q \varepsilon_s A^2) \\ &= 2 / (2 \times 10^{23} \times 1.6 \times 10^{-19} \times 12 \times 8.85 \times 10^{-14} \times 10^{-8} \text{ cm}^2) \\ &= 6 \times 10^{15} \text{ cm}^{-3} \end{aligned}$$

$$\phi_{bi} = \frac{kT}{q} \ln \frac{N_h N_l}{n_i^2} \Rightarrow N_h = \frac{n_i^2}{N_l} e^{\frac{q\phi_{bi}}{kT}} = \frac{10^{20}}{6 \times 10^{15}} e^{\frac{0.84}{0.026}} = 1.8 \times 10^{18} \text{ cm}^{-3}$$

- Is this an accurate way to determine  $N_l$ ?  $N_h$ ?

# Summary

- Concepts:
  - Joining different materials together (p & n or semiconductor and metal)
  - Two terminal devices, diode, solar-cell, LED (laser), schottky diode, protection circuits (LV), blocking (HV)
  - Some physics including neutral regions, depletion, poisson's equation
- Reading for tomorrow
  - 4.6-9