



## Problems for Seminar 2

Check the canvas page of the course for information on how seminars are run and what you are expected to do before and during the seminars.

The seminar starts with a test. The problem will be about solving a system of linear equations or converting a matrix to row echelon form.

In the seminar, the following problems will be discussed.

**Problem 1.** For each number  $a$ , we have a system of equations in three variables  $x$ ,  $y$ , and  $z$ , given by

$$(\star) \quad \begin{cases} (a-3)y = 1, \\ 2x - ax + ay - 3y + 2z - az = 1, \\ (4-2a)x + (2a-6)y + 5z - 2az = 3. \end{cases}$$

Show that the system of equations  $(\star)$  has a unique solution if and only if  $a \neq 2$  and  $a \neq 3$ . Solve the system  $(\star)$  for  $a = 2$  using row operations on the matrix for the system.

**Problem 2.** Consider the system of linear equations

$$(1) \quad \begin{cases} 17x - 13y + 2z - 7w = 5, \\ 13x + 6y - z + 11w = 3. \end{cases}$$

- (a) Determine a solution for the system when  $x = 0$  and  $w = 1$ .
- (b) Explain why the system (1) has infinitely many solutions.
- (c) Does there exist a solution to the system when  $y = -2x$  and  $w = -3x$ ?

**Problem 3.** Let

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}.$$

- (a) A matrix  $S$  is said to be a *square root* of a matrix  $M$  if  $SS = M$ . Find two square roots of  $A$ .
- (b) How many different square roots of  $B$  can you find?
- (c) Do you think that every matrix has a square root? Explain your reasoning.

*from Anton-Busby, Ex. 3.1.D6*

### MISCELLANEOUS

Here are some other topics that are important and interesting to discuss.

- Comments on Seminar 1: Anything that was unclear (theory, grading)?
- How does Gauss-Jordan elimination affect the solution set? What is the purpose of Gauss-Jordan elimination?
- What is the connection between solutions of inhomogeneous systems of equations and solutions of the associated homogeneous system?
- Pivots cannot be zero, otherwise one has to perform a change of rows. What happens if a pivot is not zero but very small? Is it okay to change rows even if the pivot is nonzero? Can this be advantageous?