Department of Mathematics

SF1626/SF1686
Several Variable Calculus

Reading suggestions

This note is intended for students that prefer to have a fast access to central parts of the material in the course and not being forced to read the whole texts in each section suggested by the course syllabus. I will, to some extent, point out these central parts in our text book: *Calculus by Adams and Essex (9th edition)*. You may complete this note with the seminar texts, suggested in each seminar.

The below suggested reading does not cover material for higher credits. For higher credits, please follow the syllabus and sections suggested in the course material.

CHAPTER 10

Section 10.1:
- Everything should be read and all examples should be done.
- Very important to be able to draw geometric figures in 3-dimensions.
- Learn carefully the topological concepts such as: Neighbourhood, open and closed sets, boundary, complement...
- Very important to be able to draw 2-dimensional domains of type

$$D := \{y > x, \quad y < x^2\} \qquad etc.$$

Section 10.5: This section is usually not taken up by the lecturer but needs to be learned. Teacher usually consider this as prerequisites so you need to know it before starting the class. Part of this section is already in your SF1624, SF1625 lectures. Therefore read it carefully and do all examples and some of the exercises. Ni will see that several exercises in this course take for granted that you are confident with this section. Try to look into your algebra course, the section of quadratic surfaces, that will be used later in the course. Learn and repeat carefully.

Section 10.6:
- Cylindrical coordinates at page 604.
- Start with Section 8.5 (pages 487–488) polar coordinates as repetition.
- Discussion at page 604 and also 606 can for a first reading be left out. Read it only if you have time.

- Example 1–3 should be carefully read.
- Page 606 Definition of spherical coordinates is very important.
- Page 607, try Example 4a.
- Ex. 4b is little harder and needs the relation between spherical and cylindrical coordinates.
- **Important:** Cylindrical and Spherical coordinates is used later for dubble and multiple integrals. So you shall need to repeat them later.

CHAPTER 11

Section 11.1:
- Definition of position vector, velocity $\mathbf{v}$ (it is a vector), speed $|\mathbf{v}|$ (the length of the velocity vector), and also the acceleration.
- Try all examples on pages 630–632.
- Look through Theorem 1 and the formulas in it.
- Do Example 6 at page 634.

Section 11.2:
- Read Example 2 page 637.

Section 11.3:
- Read carefully through the section, until and including page 647.

CHAPTER 12

Section 12.1:
- Focus first on functions of 2-variables. Try to draw pictures, and graphs. Use a graphic program, that you may find online for free, e.g., www.wolframalpha.com.
- Try first yourself and then use a program.
- Level curves should be learned. First draw the level curves yourself and the use computer to draw it, and compare.
- All example sin this section needs to be learned.
- Pages 683-684 suggests how to use maple. You will not need it if you use directly wolfram.

Section 12.2:
- Definition 2 needs to be learned to build up an idea of the concept.
- Example 2 is an easy illustration of finding limits. But to make a rigorous proof, using the definition, will be quite hard.
- Read example 3, 4, which gives you a picture of how functions can have strange behaviour, concerning their limits.
- Definition 3 (continuity) is a very important concept.
- Try with examples that shows discontinuity, rather than continuity. Or examples where the limit does not exist.
- Observe that finding the limit for a function is much harder than showing the limit does not exist.

Section 12.3:
- Partial derivatives is a central part in te course. Definition 4 page 690 needs to be learned carefully. Read pages 690–695 in detail with all examples.
- Tangent plane and normal line (pages 693–695) is very typical and comes up often as a question.

– Example 8 is an application of normal line to surfaces. You can come back to this later if you are short of time. But you need to learn it.

Section 12.4:
– Derivative of higher orders needs to be learned. Specially up to order 2.
– Red page 697, and example 1, 2 och 3.
– Theorem 1 is a must (but no prof). It says that under the conditions of the theorem the order of differentiation for higher derivatives does not matter: $f_{xy} = f_{yx}$ if the conditions of the theorem are fulfilled.

Section 12.5:
– This section is about chain rule in higher dimensions, and is a difficult section. Hence more time is needed for this.
– In a first round you can skip the Homogeneous functions and Theorem 2 (pages 708-709) and Example 10.
– You need first try the easy examples and exercies and Example 2, 3, and exercises with derivatives of first order.
– The blue/purple coloured box should be read in connections with the examples to be understood better.
– The harder examples (Exempel 3, 8, 9) are a 2-step problem, where composition of two functions are used. You should try them as well.

Section 12.6:
– Linear approximation is extremely important. Differentiability and differentials are slightly difficult concepts and you may skip it in the first round.
– In the first round go through Example 1, 3, 4.

Section 12.7:
– This section is also very important and everything needs to be read carefully. Besides examples in the book try to look at old exams where similar questions appears.

Section 12.8:
– This is an extension and application of Section 12.5 and is about implicit functions
– If you have time and the ambition do mostly examples.
– Read Example 1,3, and similars in Exercises 1–5 on page 743.
– The more ambitious student can read the system f equations on page 735–737.
– The Jacobian, Definition 8 on page 739, should be learned. You need it specially later for the change of variables in multiple integrals.
– The formula in blue/purple box, page 742 is important, specially for later use in change of variables.

Section 12.9:
– Read first examples after hearing the lecturer. The text and notations here are not optimal for self reading. So you may need to read it several times.
– Example 2, 3 gives a god picture of what is being done in this section.
– Example 4 is somewhat harder as it uses implicit functions. Simpler versions of this type of problems has occurred in A section of the exam, so it is a reading must, but at a simpler level.

CHAPTER 13

Section 13:1
- Extrem value problems are very important applications of differentiations.
- Theorem 1, page 746, is important to understand.
- Also Theorem 2 is important for understanding the topic here. You need not to read the proofs but need to know how the theorem is applied.
- Read all Examples, and also the Remark at page 757, as they are important.

Section 13:2
- Read only Example 1,2,3

Section 13:3
- For grade pass read in details pages 766–770, and Example 4. For hogher grades read the rest of the section.

Section 13:4
- Lagrange multiplier test in higher dimensions; this is a difficult section that appears for even also at lower grade level.

CHAPTER 14

Section 14.1:
- This section is an introduction to double integrals and gives no tools for computations.
- You need however to read it to understand the concept of double integrals.
- I suggest that you listen carefully to your lecturer in case you do not have time to read the section.

Section 14.2:
- Important to understand the x-simple and y-simple domains. See the pictures at page 821, and read the text there.
- Have a look at Theorem 2, and start directly with examples: Example 1–3.

Section 14.3:
- For grade pass you need in principle read examples, only.

Section 14.4:
- Double integrals in polar coordinates very useful and often appears in examples.
- Everything needs to be read carefully.
- Put an extra time on change of variables, pages 837-839, and Theorem 4. Do not forget the ABSOLUTE VALUE in the theorem on the right hand side.

Section 14.5:
- Here we have tripple integrals, but mostly simple cases.
- Our experiences has shown that many students have problems already at this stage. Therefore the section should be read carefully.

Section 14.6:
- Every single line should be read and practiced.
- Change of variables and spherical and cylindrical coordinates are very central to the course.

Section 14.7:
- Applications of what you have learned so far.
- Read carefully surface area pages 856-857.
- Read about center of mass.

– Formulas at page 859 needs to be learned.

CHAPTER 15

Section 15.1:
– Pages 867-868 should be read for the definition and concept of vector field.
– Read examples 1, 3, 4, 5.
– Observe that there are several names for line of forces (integral curves, trajectories, ...)

Section 15.2:
– The definition of conservative vector field and potential functions are central part in the course.
– The blue/purple text at page 876 are important.
– Read all examples. Example 5 (in various formulations) needs special attention.
– Pages 880–881 are self study material for those interested.

Section 15.3:
– Page 883 is definition for line integrals and needs to be learned.
– Read pages 884–886. Blue/purple text at page 884 needs to be learned.

Section 15.4:
– The definition of work needs to be learned; see page 888 the blue/purple text

$$W = \int_C \mathbf{F} \cdot d\mathbf{r}.$$

– Read Examples 1,2.
– The definition of connected and simply connected and independent of path are very important concepts, specially to simplify some computations in exercises, or exam questions.
– At page 892-893 you should learn the blue/purple coloured text. Then look at examples, 3,4.

Section 15.5:
– This is also a difficult sections. Start with learning how the normal to a surfaces can be found (see the blue/purple text at page 899 and the picture at page 899).
– In a nutshell, one can say that this is about finding dS in the integral.
– Start with computing the area of surfaces
– To have an extra function $f(x, y, z)$ is not a good idea in the beginning.
– Even if this type of question has not appeared at A part of the exam, it is very important that you can handle this type of questions as you will need it for other courses, later.
– Start with the examples to have a grip on how dS is found in different situations.
– Page 904, The attraction of a spherical shell can be skipped in a first round reading and for grade Pass.

Section 15.6:
– This again is a difficult section
– Notation $\hat{\mathbf{N}}$, is slightly confusing, so put an extra effort on it. Often this is confused with $\mathbf{n}$!

- To be able to read this section, you need to master the section before about surfaces.
- All examples are important and should be read. Questions of this type appear mostly on B and C part.
- Observe that $\mathbf{S}$ in $d\mathbf{S}$ is a vector.

CHAPTER 16

Experiences tell us that even if this is a difficult chapter one can still obtain partial points and credits from this chapter by learning the very basics and definitions, but you have to be very exact in this matter. Besides, this chapter is the most interesting part in the course with applications that also comes up in other courses.

Section 16.1: Pages 914–916 (not Theorem 1) should be read. Read Ex. 5 on page 920.

Section 16.2: Read page 923, and Ex. 1 page 926. Parts of Theorem 3: (g) and (h) you need to learn. Theorem 4, 5 you need to know but not the proof.

Section 16.3: Everything should be read several times. How the theorems are used in examples is very important.

Section 16.4: Read formula in theorem 8, and apply it directly to Examples 1–5.

Section 16.5: Stokes theorem (Theorem 10) appears often in applications. You shuld know the formula but not the proof. Read Example 1,2.