SF 2720 -Homework 1 - due (in class or via email) 18. 9. 2018

1. Exercises 2.5 in the textbook, Exercise 2.5.

2. Exercises 2.5 in the textbook, Exercise 2.8.

3. Suppose that $f : X \to X$ and $g : Y \to Y$ are topologically semi-conjugate via a continuous onto map $\phi : X \to Y$. We say $g : Y \to Y$ is a *topological factor* of $f : X \to X$.

   Prove that if $f : X \to X$ is topologically transitive then $g : Y \to Y$ is topologically transitive. Does the converse implication hold? Justify your answer.

4. For $\alpha \in \mathbb{R}$, the map F_α (often called the skew-shift map) of the 2 dimensional torus $\mathbb{T}^2$ is:

$$F_\alpha : (x, y) \mapsto (x + \alpha \mod 1, x + y \mod 1).$$

   Observe that $R_\alpha : x \mapsto x+\alpha \mod 1$, is a topological factor of F_α via the map $\phi : (x, y) \mapsto x$.

   a) Find necessary and sufficient condition on α which guarantees that F_α is topologically transitive. Is F_α minimal?

   b) Is F_α topologically mixing?

   c) Is F_α an isometry (i.e. does F preserve the distance on the torus which is induced by the Eucledian distance in $\mathbb{R}^2$)?

5. Exercises 3.5 in the textbook, Exercise 3.9.

6. An integer 2 by 2 matrix A of determinant 1 or -1 induces an invertible map on the 2-torus $\mathbb{T}^2$ which we denote by F_A. If the $\det A = 1$ then F_A is orientation preserving, and if $\det A = -1$ then then F_A is orientation reversing. The map F_A is called *toral automorphism*. F_A is called *hyperbolic* toral automorphism if for every eigenvalue λ of A we have $|\lambda| \neq 1$. Consider the following matrices with integer entries:

$$a)\ A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad b)\ A_2 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad c)\ A_3 = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \quad d)\ A_4 = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$$

   a) For each of the matrices above say if the corresponding map F_{A_i} $(i = 1, 2, 3, 4)$ is:

   - a (orientation preserving or reversing) toral automorphism? Justify your answer.

   -a hyperbolic toral automorphism? Justify your answer.

   b) Draw geometrically (as we did in class for the CAT map) how A_1 and A_4 transform the unit square. Prove or disprove topological transitivity and mixing properties for F_{A_1}, and for F_{A_4}.

   c) In the class we called a dynamical system chaotic if it has a dense set of periodic points, if it is topologically transitive and has a sensitive dependence on initial conditions. Is either of the two maps F_{A_1} and F_{A_4} a chaotic map on the 2-torus?