KTH Teknikvetenskap

# SF2719/SF2725 The History of Mathematics
# Exam
# Friday, October 26, 2018

Time: 08:00–13:00
Allowed aids: Swedish-English and/or English-Swedish dictionary
Lecturer: Tilman Bauer
Examiner: Tilman Bauer

This exam consists of three parts, each giving up to 12 points. The grade requirements are:

| | E | D | C | B | A |
|---|---|---|---|---|---|
| minimum points | 18 | 21 | 24 | 27 | 30 |
| of which on part A at least | 4 | 4 | 4 | 4 | 4 |
| and on part B at least | 4 | 4 | 4 | 8 | 10 |
| and on part C at least | 4 | 4 | 4 | 8 | 10 |

Students achieving at least 4 points on every part, but only 16 or 17 points in total, obtain the grade Fx with the possibility of completion to grade E.

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## PART A

Answer the following questions briefly. Every question gives up to 2 points. Questions 1 through 4 can be replaced by the in-class quizzes 1 through 4, respectively. If you answer one of these question and at the same time got points on the corresponding quiz, the maximum score will be taken.

**1.** What does "quadrature of the circle" mean?

**2.** Who was first to give a solution to a general polynomial equation of degree 4? Around what time did this happen?

**3.** When and by whom was it first discovered that integration and differentiation are inverse processes?

**4.** Name three mathematicians (other than Fermat, with rough dates) that made important contributions to Fermat's last theorem, along with the contributions they made.

**5.** Early hints of probability theory make an appearance in Rabbinic writing. Give an example of a problem they might study.

**6.** Which problem led Fermat to consider and prove his "little theorem"?

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*please turn over*

## PART B

This part consists of an analysis of an original text (or a translation thereof). Your bonus points from the homework essays 1 and 3 are added to the score achieved on this part. However, the total score cannot exceed 12.

**7.** Analyze the following translation with respect to the questions:

- In modern terms, what is the definition of a function given in this text?
- How does this definition differ from earlier and from later definitions?
- Based on these differences, around what time period might this text have been written?

> Because this difference between constant and variable quantities is best illustrated by an example, let us consider the flight of a ball expelled from a cannon by the force of gunpowder, since this example seems especially suited to clarifying the matter. There occur here, then, several quantities, of which the relationship is to be found in this investigation: first, of course, the quantity of gunpowder; then the elevation of the cannon above the horizon; third, the length of the flight over the horizontal plane; fourth, the time in which the expelled ball is turned in the air. And above all, unless experiments are carried out with the cannon, its length and the weight of the ball must be brought into the calculation. Truly here we must take our mind off the variety of cannons and balls, lest we fall into far too complicated questions. And therefore keeping the quantity of powder always the same, [v] as the elevation of the cannon is continually changed, there is required the length of flight with the time passed in the air by the ball; in this question the amount of powder or force of expulsion will be constant quantities, while the elevation of the cannon, and the length of flight and its duration must be regarded as variable quantities, since for all degrees of elevation we wish to define these things, that hence it will be known, how great are the changes in length and duration of flight arising from all variations in elevation. While another question will be if, keeping the elevation of the cannon the same, the quantity of gunpowder is continually changed, and we need to define the changes that hence affect the flight: for here the elevation of the cannon will be constant, while the quantity of gunpowder, and the length and duration of the flight are variable quantities. Thus it is clear, therefore, how by changing the nature of the question, it is possible to count quantities only amongst the constants or only amongst the variables. At the same time moreover, it is to be understood, what in this matter is most wanted, how variable quantities depend on each other, so that changes in one necessarily bring about changes in the others. First it is taken to be the case that the quantity of gunpowder [vi] is held the same, and by a change in elevation of the cannon the length and duration of flight are both changed; and therefore the length and duration of flight are variable quantities depending on the elevation of the cannon, changes in the latter showing at once certain definite changes [in the former]. But in the next case, those things depend on the quantity of gunpowder, a change in which must produce definite changes in those. Moreover, the quantities that depend in this way on others, so that the latter having changed, they themselves also undergo change, are usually called functions; which name opens up most generally all the ways in which one quantity may be determined from others involved with it. If therefore $x$ denotes a variable quantity, then all quantities which in any way depend on $x$, or are determined by it, are called functions of it; of this kind are the square of it, $xx$, or any other power whatever, and also quantities in any way composed from these; but also transcendental quantities, of any kind depending on $x$, such that by increase or decrease in $x$ they themselves are changed.

## PART C

This part consists of an essay. It can be replaced by the accumulated points of homework essays 2 and 4. If you write an essay here and at the same time have points from homework essays 2 and/or 4, the maximum score will be taken.

Choose **one** of the following topics and treat it in an essay. The discussion of each topic must be based on or illustrated by concrete and specific examples.

**8.** Has the pursuit of rigor helped or hindered the creation of new mathematics?

OR

**9.** Describe and assess the influence of Euclid's Elements on the development of mathematics in post-medieval Europe.

OR

**10.** Describe the development of early ideas of calculus before Newton and Leibniz.