Department of Mathematics

SF1624
Algebra and Geometry
Year 2017/2018

Problems for Seminar 3

Check the canvas page of the course for information on how seminars are run and what you are expected to do before and during the seminars.

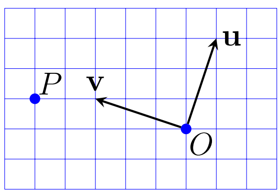
Start by doing the simpler recommended exercises from the book:

1.2: A2, A4, A5
2.3: A1, A2, A3, A4, A5, A7
4.3: A1
4.4: A1, A2

The seminar starts with a test. The problem will be about finding a basis for a subspace of $\mathbb{R}^n$ and to express a vektor with respect to that basis.

In the seminar, the following problems will be discussed.

Problem 1. The figure below shows a basis $B = \{\mathbf{u}, \mathbf{v}\}$ of $\mathbb{R}^2$ and a point P. Determine the coordinates of P with respect to the basis B.



Problem 2. Consider the matrix

$$A = \begin{bmatrix} -1 & -1 & 0 & 1 \\ -3 & -1 & 1 & 1 \\ 2 & 0 & -1 & 0 \end{bmatrix}.$$

(a) Determine a basis for the nullspace, $\mathrm{Null}(A)$.
(b) Determine a basis for the column space, $\mathrm{Col}(A)$.

Problem 3. We are given four vectors in $\mathbf{R}^4$:

$$\vec{a} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ 0 \\ 3 \\ 2 \end{bmatrix}, \quad \vec{c} = \begin{bmatrix} 0 \\ -1 \\ -2 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{d} = \begin{bmatrix} 0 \\ 0 \\ 7 \\ 2 \end{bmatrix}$$

(a) Find a basis $\mathcal{B}$ for the subspace W of $\mathbf{R}^4$ spanned by these four vectors.

(b) Is $\vec{e} = \begin{bmatrix} 3 & 0 & 6 & 4 \end{bmatrix}^T$ an element of W? If so, find the coordinate vector of $\vec{e}$ with respect to the basis $\mathcal{B}$.

(c) Extend the basis $\mathcal{B}$ to a basis $\mathcal{B}'$ of all of $\mathbf{R}^4$.

(d) Compute the coordinate vector of $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ with respect to the basis $\mathcal{B}'$.

Problem 4. Which of the following sets are vektor spaces? Find a basis and the dimension for those that are.

(a) All vectors $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$ i $\mathbf{R}^4$ such that $x + y + z - w = 1$

(b) All polynomial functions $f \colon \mathbb{R} \to \mathbb{R}$ of degree ≤ 5 (i.e. $f(x) = a + bx + cx^2 + dx^3 + fx^4 + gx^5$)

(c) All invertible 3×3-matrices

(d) All 3×3-matrices that satisfy $A^T = -A$. Here, A^T denotes the transpose matrix of A.

MISCELLANEOUS

Here are some other topics that are important and interesting to discuss.

- What is the definition of a vector space? Which examples are there besides $\mathbb{R}^n$?
- What does it mean to be linearly independent, a linear span, and a basis? What is the definition of dimension?