Department of Mathematics

SF1624
Algebra and Geometry
Year 2017/2018

Problems for Seminar 2

Check the canvas page of the course for information on how seminars are run and what you are expected to do before and during the seminars.

Start by doing the simpler recommended exercises from the book:

1.1: A1, A5, A8, A9
1.3: A2, A5, A7, A9
1.4: A1, A5, A6
1.5: A1, A3, A4, A5, A6

The seminar starts with a test. The problem will be about computing distances between points, lines, or planes.

In the seminar, the following problems will be discussed.

Problem 1. The points P, Q, and R have coordinates

$$P = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad Q = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad R = \begin{bmatrix} 4 \\ 6 \\ -1 \end{bmatrix},$$

and the plane Π is given by the equation

$$2x - y + 2z = 3.$$

(a) Compute a parameter form for the line L through P and Q.
(b) Does the line L contain the point R?
(c) Determine the intersection of Π and L.
(d) Compute an equation for the plane which is orthogonal to L and contains P.

Problem 2. For each number t, we are given a triangle T with vertices

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} t \\ t \\ 1 \end{bmatrix}.$$

(a) For which value of t is T right-angled at A?
(b) Compute the other two angles for this value of t (use a calculator).
(c) Compute the area of T for $t = 3$.
(d) Find a value for t such that the point C is closest to A.

Problem 3. The line L_1 passes through the points A_1 och B_1, and the line L_2 through the points A_2 och B_2, where

$$A_1 = \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} -3 \\ 6 \\ 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \quad B_2 = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}.$$

(a) Find a parameter form and an equation for the plane Π which contains L_2 and is parallel with L_1.
(b) Explain why the shortest distance d between the lines L_1 and L_2 equals the distance between the plane Π and the point A_1, and compute it.
(c) Compute the image P of A_1 under the orthogonal projection onto the plane Π.
(d) Find numbers λ and μ such that $P = A_2 + \lambda(B_1 - A_1) + \mu(B_2 - A_2)$. Show that the points $C_1 = A_1 + \lambda(B_1 - A_1)$ and $C_2 = A_2 + \mu(B_2 - A_2)$ are points on L_1 and L_2, respectively, and that they have distance d from each other.

Problem 4. The points A, B, C, D are given by

$$A = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \quad C = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \quad D = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

(a) Find a normal vector to the plane Π_1 which contains the origin, A, and B; and a normal vector to the plane Π_2 containing B, C, and D.
(b) Find a parameter form for the line of intersection of Π_1 and Π_2 using the cross product of the normal vectors.
(c) Spell "*parallelepiped*".
(d) Compute the volume of the parallelepiped spanned by $\vec{AB}$, $\vec{AC}$, $\vec{AD}$.

MISCELLANEOUS

Here are some other topics that are important and interesting to discuss.

- Comments on Seminar 1: Anything that was unclear (theory, grading)?
- How many equations does one need to define an m-dimensional subspace of $\mathbb{R}^n$? How many free variables are needed in a parametric form?
- What is the relationship between projections and the notion of distance?
- What is the expected intersection between two planes in $\mathbb{R}^3$, in $\mathbb{R}^4$, and in $\mathbb{R}^5$?