



Problems for Seminar 1

Check the canvas page of the course for information on how seminars are run and what you are expected to do before and during the seminars.

Start by doing the simpler recommended exercises from the book:

2.1: A3,A5

2.2: A3

The seminar starts with a test. The problem will be about a Gauss-Jordan elimination, transformation to row echelon form, or the solution of a linear system of equations.

In the seminar, the following problems will be discussed.

Problem 1. For each number a , we have a system of equations in three variables x , y , and z , given by

$$(\star) \quad \begin{cases} (a-3)y = 1, \\ 2x - ax + ay - 3y + 2z - az = 1, \\ (4-2a)x + (2a-6)y + 5z - 2az = 3. \end{cases}$$

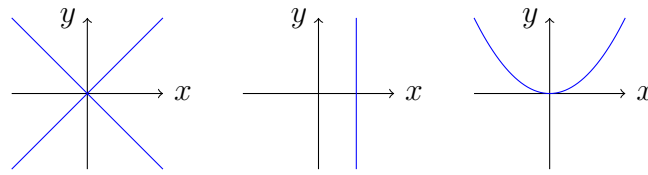
Show that the system of equations $(\star)$ has a unique solution if and only if $a \neq 2$ and $a \neq 3$. Solve the system $(\star)$ for $a = 2$ using row operations on the matrix for the system.

Problem 2. Consider the system of linear equations

$$(1) \quad \begin{cases} 17x - 13y + 2z - 7w = 5, \\ 13x + 6y - z + 11w = 3. \end{cases}$$

- (a) Determine a solution for the system when $x = 0$ and $w = 1$.
- (b) Explain why the system (1) has infinitely many solutions.
- (c) Does there exist a solution to the system when $y = -2x$ and $w = -3x$?

Problem 3. Consider the subsets U , V , and W of $\mathbb{R}^2$ given by the equations $(x+y)(x-y) = 0$, $2x = 1$, and $y = x^2$, respectively.



- (a) Define what it means to be a subspace of $\mathbb{R}^2$.
- (b) Explain, using the definition, why none of the three given subsets U , V , or W are a subspace of $\mathbb{R}^2$.

MISCELLANEOUS

Here are some other topics that are important and interesting to discuss.

- What is the meaning of the terms solution set, subset, and subspace? What is Euclidean n -space?
- How does Gauss-Jordan elimination affect the solution set? What is the purpose of Gauss-Jordan elimination?
- What is the connection between solutions of inhomogeneous systems of equations and solutions of the associated homogeneous system?
- Pivots cannot be zero, otherwise one has to perform a change of rows. What happens if a pivot is not zero but very small? Is it okay to change rows even if the pivot is nonzero? Can this be advantageous?