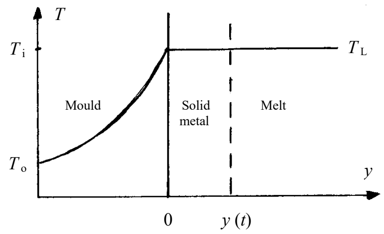
Casting Processing, MH2252, 6hp



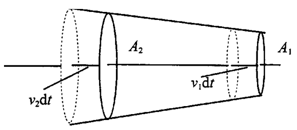
Lecture 4a+b
Heat Transport during Casting and Solidification
Sand mould casting – Good contact

Today´s topics

- Repetition
  - Casting Hydrodynamics
- Heat Transport during Casting
  - Basic Concepts
  - Modes of Solidification
- Theory of Heat Transport in Casting
  - Ideal Contact Model
  - Sand Mould Casting

Basic Hydrodynamics

- Continuity Principle
  - Incompressible Liquid



$$v_1A_1 = v_2A_2$$

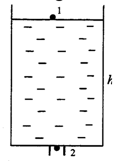
- No fluid appears or disappears during the flow
- The fluid flow volume is constant by time

Used whenever you have a change of area!

Basic Hydrodynamics

- Bernoulli's Equation
  - Laminar flow

Teeming a ladle



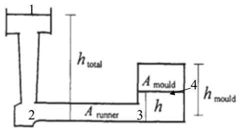
$$p_1+\rho g h_1+\frac{\rho v_1^2}{2}=p_2+\rho g h_2+\frac{\rho v_2^2}{2}$$

- Torricelli's law: $v=\sqrt{2gh}$
  ($A_1>>A_2 \rightarrow v_1<<v_2 \rightarrow v_1 \approx 0$ (Continuity Principle)
-)
- The first part: Pressure work
- The second part: Potential energy
- The third part: Kinetic energy

Used whenever you have a difference in height (change of potential energy)

Lect.4-4

Hydrodynamics at Uphill Casting



$v_2=\sqrt{2g(h_{tot}-h)}$ — Melt velocity at point 2, from Bernouilli's eq. The same area and velocity in point 2 and 3 by Continuity Principle, $v_2 = v_3$

$A_m dh = A_i\sqrt{2g(h_{tot}-h)}\cdot dt$ — Height (volume) change, *dh* , by the time *dt*, at point 4 (v_4), by combining with the Continuity principle at point 3

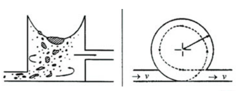
$\frac{A_i}{A_m}\int_0^{t_f} dt = \frac{1}{\sqrt{2g}}\cdot\int_0^{h_m}\frac{dh}{\sqrt{h_{tot}-h}}$ — Integrate, to get the filling time, t_f

$t_f = \frac{2A_m}{A_i\sqrt{2g}}\cdot\left(\sqrt{h_{tot}}-\sqrt{h_{tot}-h_m}\right)$ — The filling time, t_f by mould area (A_m), inlet area (A_i), sprue height (h_{tot}) and mould height (h_{mould})

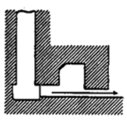
Lect.4-5

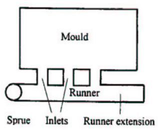
Inclusion Control

- Swirl Trap

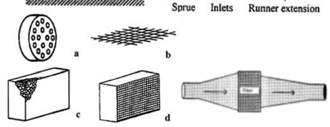


- Mechanical impurity trap and Runner Extension





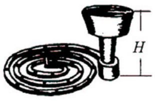
- Ceramic Filters

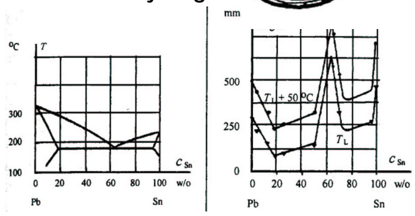


Lect.9-6

Maximum Fluidity Length

- Spiral for check of the maximum fluidity length

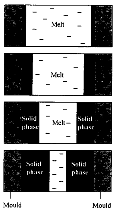




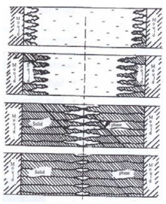
Maximum fluidity length is found for pure metals and eutectic compositions in the system Pb-Sn

Lect.3-7

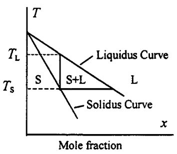
Note: Modes of Solidification will affect the maximum fluidity length!



- Pure metal or eutectic alloy
- Alloy with small ΔT and strong cooling



- Alloy with large ΔT
- Weak cooling



Solidification interval
$\Delta T = T_L - T_S$

Lect.4-8

Heat Transport

- Thermal Conduction
- Thermal Radiation
- Convection

Casting of metals is closely related to;
a) heat release ($-\Delta H$) at solidification and
b) heat transport and cooling.

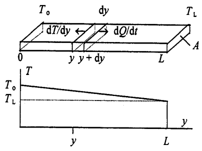
The rate of heat removal is very important as it among others decides the solidification time and the structure formation of the casting

Lect.4-9

Thermal Conduction

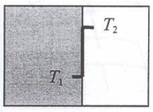
- Fourier's 1st Law (one dimension)

$$\frac{\partial Q}{dt} = -k A \frac{\partial T}{\partial y}$$



The temperature is a function of position but not of time (stationary conditions).
k = thermal conductivity

Conduction across an Interface



$$\frac{\partial Q}{dt} = -hA(T_2 - T_1)$$

$\frac{\partial Q}{dt} = heatflow$ across interface

h = heat transfer coefficient
A = surface area
T = temperature

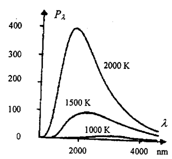
$$h = \frac{k}{\delta}$$

δ = width of interface
k = thermal conductivity
h = heat transfer coefficient

Radiation

$$\frac{\partial Q}{dt} = -\varepsilon \sigma A \left(T_1^4 - T_2^4\right)$$

ε = emissivity
σ = Stefan Bolzmann's constant



Perfect Black Body

Material	*T* °C	ε
Al-film	100	0.09
Oxidized Al	150 - 500	0.20 - 0.30
Polished steel	100	0.066
Cast iron	22	0.44
Cast iron	880 - 990	0.60 - 0.70
Low carbon steel	230 - 1065	0.20 - 0.32
High carbon steel	100	0.074

All bodies emit electromagnetic radiation or *heat radiation* to their surroundings

Convection

- Natural convection (free convection)
- Forced convection

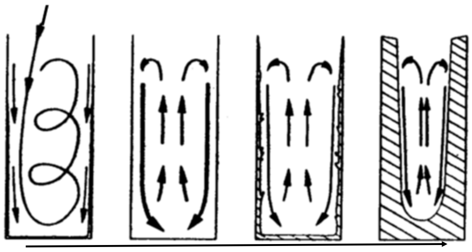
$$\frac{\partial Q}{dt} = -h_{conv} A(T - T_0)$$

Table 3. Heat transfer numbers of convection in some specified cases.

Specifications	$h_{c\ con}$ W/m²K
Free Convection	
Horisontal cylinder with radius 2.5 cm in air.	6.5
Horisontal cylinder with radius 1.0 cm in water.	890
Forced Convection	
Air beam with the velocity 2 m/s over a square plate (0.2 m)².	12
Flowing water with the "velocity" 0.5 kg/s in a tube with with a diameter of 2.5 cm.	3 500

Lect.4-13

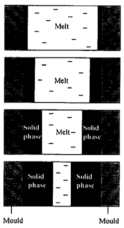
Convection



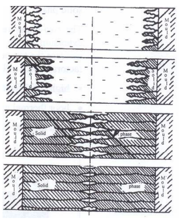
- Natural convection (free convection). Normally driven by differences in liquid density

Lect.4-14

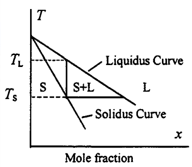
Modes of Solidification – influence of cooling and composition



- Pure metal or eutectic alloys. Alloys with small ΔT and strong cooling



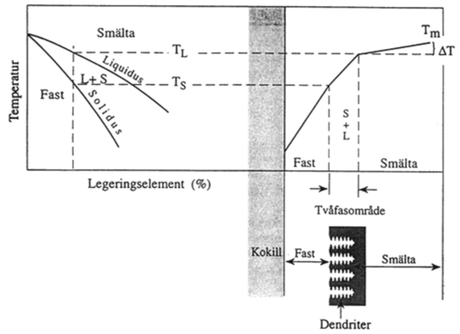
- Alloys with large ΔT and weak cooling



Solidification interval $\Delta T = T_L - T_S$

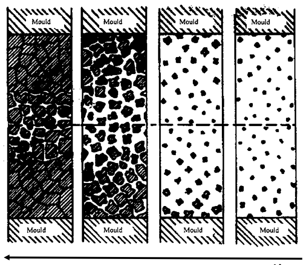
Lect.4-15

Modes of Solidification – dendritic growth



• Relation between composition (solidification interval), cooling rate and dendritic structure

Modes of Solidification – nodular growth



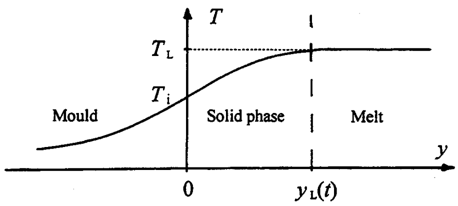
Alloys with broad solidification interval and/or solidification in a sand mould (weak cooling).
The local solidification occurs within a constant liquid/solid sphere and not by a planar solidification front

Heat Transfer: Special Cases Useful in Casting

- Ideal Contact Metal-Mould
  - Liquid Metal – Mould
- Good Contact Metal-Mould
  - Sand Mould Casting
- Poor Contact Metal-Mould
  - Permanent Moulds
  - Continuous Casting

Ideal Contact Metal-Mould

Temperature distribution



See chapter 4.3.2 Theory of Heat Transport at Casting with Ideal Contact between Metal and Mould

Lect.4-19

Ideal Contact Metal-Mould – Model 0

$$T_{mould} = A_{mould} + B_{mould} \cdot \operatorname{erf}\left(\frac{y}{\sqrt{4\alpha_{mould} t}}\right)$$

$$T_{metal} = A_{metal} + B_{metal} \cdot \operatorname{erf}\left(\frac{y}{\sqrt{4\alpha_{metal} t}}\right)$$

$$A_{mould} = A_{metal} = T_o + \frac{T_L - T_o}{\frac{k_{metal}\sqrt{\alpha_{mould}}}{k_{mould}\sqrt{\alpha_{metal}}} + \operatorname{erf}\lambda} \cdot \frac{k_{metal}\sqrt{\alpha_{mould}}}{k_{mould}\sqrt{\alpha_{metal}}}$$

$$B_{metal} = \frac{T_L - T_o}{\frac{k_{metal}\sqrt{\alpha_{mould}}}{k_{mould}\sqrt{\alpha_{metal}}} + \operatorname{erf}\lambda}$$

$$B_{mould} = \frac{T_L - T_o}{\frac{k_{metal}\sqrt{\alpha_{mould}}}{k_{mould}\sqrt{\alpha_{metal}}} + \operatorname{erf}\lambda} \cdot \frac{k_{metal}\sqrt{\alpha_{mould}}}{k_{mould}\sqrt{\alpha_{metal}}}$$

1. Boundary Condition for the Mould

$$T(t,-\infty) = T_o$$

2. Boundary Condition for the Interface Mould/Metal

$$T(t,0) = T_i$$

3. Boundary Condition for the Interface Mould/Metal

$$k_{mould}\frac{\partial T_{mould}}{\partial y} = k_{metal}\frac{\partial T_{metal}}{\partial y}$$

4. Boundary Condition for the Solidification Front

$$T(t, y_L(t)) = T_L$$

$$\frac{dq}{dt} = (-\Delta H)\cdot\rho\cdot 1\cdot\frac{dy_L(t)}{dt}$$

5. Boundary Condition for the Metal

$$k_{metal}\left(\frac{\partial T_{metal}}{\partial y}\right)_{y=y_L(t)} = (-\Delta H)\cdot\rho\cdot\frac{dy_L(t)}{dt}$$

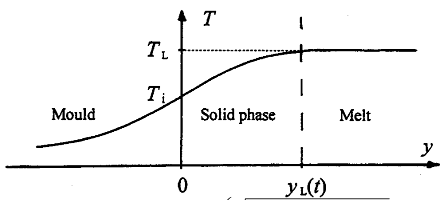
The *growth rate* or *solidification rate by the ideal model*

$$\boxed{\frac{dy_L(t)}{dt} = \lambda\cdot\sqrt{\frac{\alpha_{metal}}{t}}}$$

Lect.4-20

Ideal Contact Metal-Mould – Model 0

Find λ by iteration (guess a value), that satisfies the below equation, insert in $y_L(t)$ to describe the position of the solidification front as a function of time



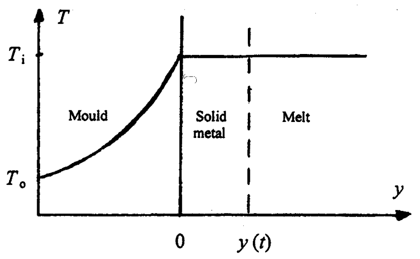
$$\frac{c_p^{metal}(T_L - T_o)}{-\Delta H} = \sqrt{\pi}\cdot\lambda\, e^{\lambda^2}\cdot\left(\sqrt{\frac{k_{metal}\rho_{metal}c_p^{metal}}{k_{mould}\rho_{mould}c_p^{mould}}} + \operatorname{erf}\lambda\right)$$

$$\boxed{y_L(t) = \lambda\cdot\sqrt{4\alpha_{metal}t}}$$

$$\alpha_{metal} = \frac{k_{metal}}{\rho_{metal}\cdot Cp^{metal}}$$

Lect.4-21

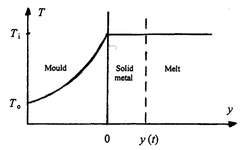
Sand Mould Casting

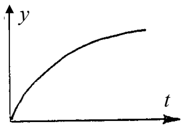


See chapter 4.4.1 Temperature Distribution at Sand Mould Casting

Lect.4-22

Sand Mould Casting





1. The conductivity of the metal is very large compared to that of the sand mould ($k_{mould} << k_{metal}$).

2. During the casting the temperature of the mould wall immediately becomes equal to the temperature T_L of the melt and keeps this temperature throughout the whole solidification process.

3. At large distances from the interface the temperature of the mould is equal to the room temperature T_o.

Lect.4-23

Sand Mould Casting – Model 1

$$T_{\text{mould}} = A_{\text{mould}} + B_{\text{mould}} \cdot \operatorname{erf}\left(\frac{y}{\sqrt{4\alpha_{\text{mould}} t}}\right)$$

$$T_{\text{metal}} = A_{\text{metal}} + B_{\text{metal}} \cdot \operatorname{erf}\left(\frac{y}{\sqrt{4\alpha_{\text{metal}} t}}\right)$$

$$A_{\text{mould}} = A_{\text{metal}} = T_o + \frac{T_L - T_o}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} + \operatorname{erf}\lambda} \cdot \frac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}}$$

$$B_{\text{metal}} = \frac{T_L - T_o}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} + \operatorname{erf}\lambda}$$

$$B_{\text{mould}} = \frac{T_L - T_o}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} + \operatorname{erf}\lambda} \cdot \frac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}}$$

1. At the interface (y=0) between the sand mould and metal we get ($B_{metal}=B_{mould}=0$):

$$T_i = A_{\text{mould}} = A_{\text{metal}}$$

Insert in equation to get the T_i:

$$A_{\text{mould}} = A_{\text{metal}} = T_i = T_o + \frac{T_L - T_o}{1 + \dfrac{\operatorname{erf}\lambda}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}}}}$$

2. Sand has poor thermal conductivity compared to metals, which means that $k_{metal} >> k_{mould}$

$$\frac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} = \frac{k_{\text{metal}}\sqrt{\dfrac{k_{\text{mould}}}{\rho_{\text{mould}} c_p^{\text{mould}}}}}{k_{\text{mould}}\sqrt{\dfrac{k_{\text{metal}}}{\rho_{\text{metal}} c_p^{\text{metal}}}}} = \frac{\sqrt{k_{\text{metal}}\rho_{\text{metal}} c_p^{\text{metal}}}}{\sqrt{k_{\text{mould}}\rho_{\text{mould}} c_p^{\text{mould}}}} >> 1$$

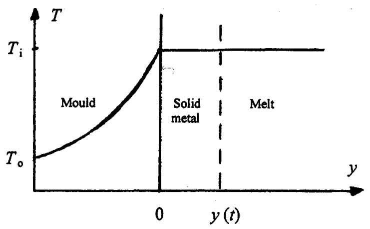
3. The value of erf λ in equation lies between 0 and 1. The second term in the denominator in the equation will thus be very small and the denominator ≈ 1

$$T_i = T_{i\ \text{metal}} = T_{i\ \text{mould}} \approx T_L$$

Lect.4-24

Sand Mould Casting – Model 1

General Assumption: $T_i = T_{i,metal} = T_{i,mould} = T_L$



$$y_L(t) = \frac{2}{\sqrt{\pi}} \cdot \frac{T_i - T_o}{\rho_{metal}(-\Delta H)} \cdot \sqrt{k_{mould}\rho_{mould}\, c_p^{mould}} \cdot \sqrt{t}$$

$$y_L(t) = \int_0^t \frac{T_i - T_o}{\rho_{metal}(-\Delta H)} \cdot \sqrt{\frac{k_{mould}\rho_{mould}\, c_p^{mould}}{\pi}} \cdot \frac{dt}{\sqrt{t}}$$

The *growth rate* dy_L/dt or *solidification rate by the Sand Mould Model*

Lect.4-25

Sand Mould Casting - Chvorinov's first assumption

$$y_L = \left(\frac{V_{metal}}{A}\right)$$

Sphere: $y_L = r/3$

Cube: $y_L = a/6$

$$\frac{V_{metal}}{A} = \frac{2}{\sqrt{\pi}} \cdot \frac{T_i - T_o}{\rho_{metal}(-\Delta H)} \cdot \sqrt{k_{mould}\rho_{mould}\, c_p^{mould}} \cdot \sqrt{t_{total}}$$

- y_L = The solidification distance
- V_{metal} = Total volume of the casting (mould cavity)
- A = Total area of the interface between the mould and the metal (cooling interface)
- t_{total} = The solidification time of the casting

Lect.4-26

Sand Mould Casting - Chvorinov's Rule

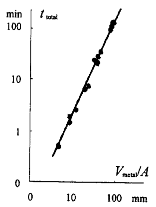
- Chvorinov's Rule
  - Every unit area of the mould absorb equal amount of heat
  - Experimentally verified

$$t_{total} = C\left(\frac{V_{metal}}{A}\right)^2$$

V=mould volume, *A*=cooling mould area. Note the power of 2!

$$C = \left(\frac{\sqrt{\pi}}{2} \cdot \frac{\rho_{metal}(-\Delta H)}{T_L - T_0} \cdot \frac{1}{\sqrt{k_{mould}\rho_{mould}c_p^{mould}}}\right)^2$$

Only material properties of the melt and the mould!

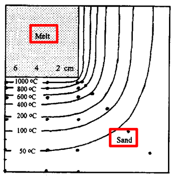


Note: Linear relation due to logarithmic diagram

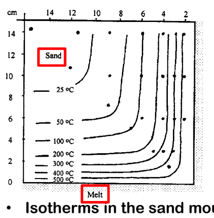
Lect.4-27

Sand Mould Casting – deviations from Chvorinov's Rule

Heat Conduction at Sharp Corners. The larger the distances between the isotherms, the smaller is the temperature gradient and the heat conduction in the sand



- Isotherms in the sand mould outside an outer corner of a casting
- More energy transport by the diagonal than the sides



- Isotherms in the sand mould inside an inner corner of a casting (melt=mould cavity)
- Less energy transport by the diagonal than the sides

Lect.4-28

Recommended reading in "Materials Processing during Casting", by Hasse Fredriksson and Ulla Åkerlind

Chapter:

- ❑ 4.1 – 4.4
- ❑ 5.1 – 5.7

Lect.4-29