Materials Processing during Casting Summary material



Allowed material at the exam in the course MH2252 Casting Processing Part II H18

Basic Hydrodynamics

Principle of Continuity: $A_1v_1 = A_2v_2$

Bernoulli's Law at Laminar Flow:

For laminar flow in metal melts Bernoulli's law is valid if friction losses are neglected

$$p_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = p_2 + \rho g h_2 + \frac{\rho v_2^2}{2} = \text{const}$$

Special case:

The exhaust velocity of the melt is determined by the height of the free surface over the bottom hole of the ladle

$$v_2 = \sqrt{2gh}$$

Gating Systems at Component Casting
 Downhill Casting



Casting Rate

Casting rates can be calculated by aid of basic hydrodynamic laws. The result depends on the type and design of the gating system.



Penetrable mould:

Impenetrable mould:



A single gate is enough for small castings. For large castings several gates are required.

Depending of the position of the gates the casting methods are characterized as

- uphill casting
- downhill casting
- casting with lateral sprue.

In the latter case special measures have to be taken to achieve an even pressure at all gates.

Casting Time

In simple cases the time to empty the ladle and the casting time can be calculated by application of the hydrodynamic laws in each special case. Empirical relation between casting time and mass at component or ingot casting:

 $t = A \cdot m \operatorname{casting}^{n}$

Uphill Casting

Penetrable Mould:

$$v = \sqrt{2g (h_{\text{total}} - h)}$$
$$t_{\text{fill}} = \frac{2A_{\text{m}}}{A_{\text{sprue}}\sqrt{2g}} \cdot \left(\sqrt{h_{\text{total}}} - \sqrt{h_{\text{total}}} - h_{\text{m}}\right)$$



The driving force of metal tranfer is an external pressure.



Gating System at Ingot Casting

The principles, formulas, position of gating systems and inclusion control are the same for component casting and ingot casting, with reservation for modifications, due to differences in size.

Casting time: See top of this page.



Thermal energy is transported by means of conduction, radiation and convection.

Heat Conduction

Heat Conduction in a Layer. Stationary Conditions

$$\frac{\mathrm{d}q}{\mathrm{d}t} = -k \frac{\mathrm{d}T}{\mathrm{d}x}$$
 Fourier's first law

k = thermal conductivity. Its SI unit is J/msK or W/mK.

Heat Transfer at the Interface between Two Different Materials. Stationary Conditions

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -hA \cdot \left(T_2 - T_1\right) \qquad \text{or} \qquad \frac{\mathrm{d}q}{\mathrm{d}t} = -h \cdot \left(T_2 - T_1\right)$$

h = heat transfer coefficient, heat transfer constant or heat transfer number. Its SI unit is J/m²sK or W/m²K.

Heat Conduction through Several Layers and Interfaces in Series. Stationary Conditions

$$\frac{L}{kA} = \frac{L_1}{k_1A_1} + \frac{L_2}{k_2A_2} + \frac{L_3}{k_3A_3} + \dots + \frac{1}{h_4A_4} + \frac{1}{h_5A_5} \dots$$

Relation between h and k at an air gap:









General Law of Heat Conduction. Non-Stationary **Conditions**

 $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2}$ Fourier's second law The constant $\alpha = \frac{k}{\rho c_{p}}$ is called *thermal diffusitivity* and is measured in the SI unit m^2/s .

Heat Radiation

 $\mathrm{d}W_{\mathrm{total}} = \varepsilon \cdot \sigma \cdot A \left(T^{4} - T_{\mathrm{o}}^{4} \right) \cdot \mathrm{d}t$ $\varepsilon = emissivity$, a dimension less factor < 1. $h_{\rm rad} = \varepsilon \cdot \sigma \cdot (T^2 + T_{\rm o}^2)(T + T_{\rm o}).$

Convection

Water cooling in an example of *forced* convection. The motion of the flowing water is controlled.

Natural of free convection occurs under the influence of gravitation forces without external affection.

$$\frac{\mathrm{d}q}{\mathrm{d}t} = h_{\mathrm{con}} \left(T - T_{\mathrm{o}} \right)$$

Т

$T_{\rm L}$ $T_{\rm S}$ Solidification process of an alloy with a broad solidification interval.

Solidification Processes in Alloys



Heat Transport at Casting with Ideal Contact between Metal and Mould

Temperature Distribution in Metal and Mould

The temperature distribution is calculated by solving the general law of heat conduction and determining the constants by aid of boundary conditions like given temperatures of the melt, of the interface mould /metal and the surroundings.



Five constants appear in the solution. One of them is λ . It is determined by means of iteration from the equation

$$\frac{c_p^{\text{metal}}(T_{\text{L}} - T_{\text{o}})}{-\Delta H} = \sqrt{\pi} \cdot \lambda \, \mathrm{e}^{\lambda^2} \cdot \left(\sqrt{\frac{k_{\text{metal}}\rho_{\text{metal}}c_p^{\text{metal}}}{k_{\text{mould}}\rho_{\text{mould}}c_p^{\text{mould}}}} + \operatorname{erf} \lambda\right)$$

When λ is known some important quantities can be calculated:

Position of the solidification front: $y_{\rm L}(t) = \lambda \cdot \sqrt{4\alpha_{\rm metal}t}$

Solidification rate:
$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = \lambda \cdot \sqrt{\frac{\alpha_{\mathrm{metal}}}{t}}$$

Heat Transport at Casting with Poor Contact between Solid Metal and Mould

At poor contact between mould and metal there is a discontinuity of the temperature at the interface. This is a very common case. When the solidifying shell solidifies and cools, it shrinks and looses the contact with the mould wall. The heat contact between mould and metal becomes suddenly poor.



Solidification Processes with a Temperature Decrease across the Interface

When the temperature of the melt, the conductivity, the heat transfer number of the air gap and the temperature of the surroundings are known, it is possible to calculate the

Heat flux at the interface:

$$\frac{\mathrm{d}q}{\mathrm{d}t} = h\left(T_{\mathrm{i metal}} - T_{\mathrm{o}}\right)$$

Temperature of the metal at the metal/mould interface:

$$T_{\rm i metal} = \frac{T_{\rm L} - T_{\rm o}}{1 + \frac{h}{k} y_{\rm L}(t)} + T_{\rm o}$$

Solidification rate:
$$\frac{dy_L}{dt} = \frac{T_L - T_o}{\rho \left(-\Delta H\right)} \cdot \frac{h}{1 + \frac{h}{k} \cdot y_L}$$

Relation between the solidification time and the thickness of the solidified layer:

$$t = \frac{\rho\left(-\Delta H\right)}{T_{\rm L} - T_{\rm o}} \cdot \frac{y_{\rm L}}{h} \cdot \left(1 + \frac{h}{2\,k} \cdot y_{\rm L}\right)$$

At unilateral cooling $y_L = s$ and the time *t* is equal to *the total solidification time*.

At symmetrical bilateral cooling $y_{\rm L} = \frac{s}{2}$, which gives a shorter total solidification time.

Influence of Vacancies and other Lattice Defects on Heat of Fusion and Heat Capacitivity

Heat of Fusion

It has been found that the heat of fusion $(-\Delta H)$ is not always constant but depends on the solidification rate or the cooling rate at the solidification process. For some alloys the observed heat of fusion $(-\Delta H^{\text{eff}})$ decreases with increasing growth rate.

$$-\Delta H_{\rm m}^{\rm eff} = \left(-\Delta H_{\rm m}^{\rm e}\right) - \left(x_{\rm vac} - x_{\rm vac}^{\rm e}\right) \cdot \left(-\Delta H_{\rm m}^{\rm vac}\right)$$

Heat Capacitivity

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A similar influence of vacancies and other lattice defects on the heat capacitivity has been observed:

$$C_{\rm p}^{\rm eff} = C_{\rm p}^{\rm e} + \frac{\mathrm{d}x_{\rm vac}}{\mathrm{d}T} \cdot \left(-\Delta H_{\rm m}^{\rm vac}\right) + \frac{\mathrm{d}x_{\rm vac}}{\mathrm{d}t} \cdot \left(-\Delta H_{\rm m}^{\rm vac}\right)$$

Heat Transport at Component Casting

Heat Transport at Casting in Sand Moulds

Temperature Distribution in a Dry Sand Mould

The contact between the sand mould and the metal is poor. However, due to the poor thermal conductivity of the sand mould, there is no discontinuity in temperature at the interface metal/mould. In this respect it appears as if the contact were ideal.

The solution of the general law of heat conduction is thus analogous to the one in case of ideal cooling. λ is determined by means of iteration from the equation

$$\frac{c_{\rm p}^{\rm metal}(T_{\rm L} - T_{\rm o})}{-\Delta H} = \sqrt{\pi} \cdot \lambda \, {\rm e}^{\lambda^2} \cdot \left(\sqrt{\frac{k_{\rm metal}\rho_{\rm metal}c_{\rm p}^{\rm metal}}{k_{\rm mould}\rho_{\rm mould}c_{\rm p}^{\rm mould}}} + {\rm erf} \, \lambda\right)$$

Calculations show that

$$T_{i} = T_{i \text{ metal}} = T_{i \text{ mould}} \approx T_{L}$$

At casting in a sand mould the temperature at the interface between the metal and the mould is approximately equal to the liquidus temperature of the melt.



In a moist sand mould a waterfront or evaporation front is formed.

Thickness of Solidified Shell

Thickness of the solidified layer is a parabolic function of time:

$$y_{\rm L}(t) = \frac{2}{\sqrt{\pi}} \cdot \frac{T_{\rm L} - T_{\rm o}}{\rho_{\rm metal} \left(-\Delta H\right)} \cdot \sqrt{k_{\rm mould} \rho_{\rm mould} c_{\rm p}^{\rm mould}} \cdot \sqrt{t}$$



Chvorinov's Rule:

At casting in a sand mould the solidification time is proportional to the square of the ratio volume/area of the casting.

$$t_{\text{total}} = C \cdot \left(\frac{V_{\text{metal}}}{A}\right)^2$$
 Chvorinov's rule

where

$$C = \frac{\pi}{4} \cdot \frac{\rho_{\text{metal}}^2 \left(-\Delta H\right)^2}{\left(T_{\text{i}} - T_{\text{o}}\right)^2 k_{\text{mould}} \rho_{\text{mould}} c_{\text{p}}^{\text{mould}}}$$

Heat Transport at Casting in Permanent Moulds

Permanent moulds are made of metal. At solidification of the casting an air gap appears between the mould and the casting, which results in poor thermal conduction.

Heat Transport across an Interface with an Air Gap

Definition of Nussel's number: N

$$u = \frac{hs}{k}$$

Temperature of the metal/mould interface if Nu << 1:

$$T_{\rm i metal} = \frac{T_{\rm L} - T_{\rm o}}{1 + \frac{h}{k} y_{\rm L}(t)} + T_{\rm o}$$

Relation between the solidification time and the thickness of the solidified layer if $Nu \ll 1$:

$$t = \frac{\rho\left(-\Delta H\right)}{T_{\rm L} - T_{\rm o}} \cdot \frac{y_{\rm L}}{h}$$

At unilateral cooling $y_L = s$ and the time *t* is equal to *the total solidification time*.

At symmetrical bilateral cooling $y_{\rm L} = \frac{s}{2}$, which gives a shorter total solidification time.



Calculation of Nussel's number is very useful to choose model for calculations on a given casting process.

If Nu << 1 the simplified equations are valid.

If the Nussel criterion is not fulfilled then the normal equations for heat transport at casting with poor contact between metal and mould are valid.

Natural Convection in Metal Melts



The temperature differences within the melt cause natural convection in ingots. Within a thin layer close to the solidification front melt moves downwards.

Temperature boundary layer:

$$T = T_{\text{melt}} - (T_{\text{melt}} - T_{\text{i metal}}) \cdot \left(1 - \frac{y}{\delta}\right)^2$$

Velocity boundary layer:

$$u = u_{o}(z) \cdot \frac{y}{\delta} \cdot \left(1 - \frac{y}{\delta}\right)^{2}$$

Maximum of the temperature boundary layer:

$$\delta(z) = 3.93 \cdot \left(\frac{v_{\text{kin}}}{\alpha}\right)^{-\frac{1}{2}} \cdot \left(\frac{20}{21} + \frac{v_{\text{kin}}}{\alpha}\right)^{\frac{1}{4}} \cdot \left[\frac{g\beta\left(T_{\text{melt}} - T_{\text{i metal}}\right)}{v_{\text{kin}}^2}\right]^{-\frac{1}{4}} \cdot z^{\frac{1}{4}}$$
or
$$\delta(z) = B \cdot \left[\frac{g}{z} \cdot \left(T_{\text{melt}} - T_{\text{i metal}}\right)\right]^{-\frac{1}{4}}$$



Nucleation

Solidification starts with formation of nuclei in the melt. The condition for this process is that the melt is undercooled.

The nucleation is homogeneous if the nuclei are formed from the melt. At heterogeneous nucleation the nuclei consist of foreign elements or substances.

Dendritic Growth

Relation between Growth Rate and Dendrite Arm Distance

 $v_{\text{growth}} \cdot \lambda^2 = \text{const}$

Relation between Growth Rate and Undercooling:

 $v_{\text{growth}} = \mu \left(T_{\text{L}} - T \right)^{\text{n}}$

Eutectic Growth

Precipitation of a eutectic alloy occurs at the constant eutectic temperature $T_{\rm E}$.



Two phases with the compositions x_{α} and x_{β} are formed. A normal eutectic lamella structure is arises when the two precipitated phases grow side by side in co-operation.



Relation between Growth Rate and Lamella Distance

 $v_{\text{growth}} \cdot \lambda^2 = \text{const}$

In several casting processes the growth occurs radially from centres. These eutectic cells grow until they collide and fill the whole volume.

Relation between Growth Rate and Undercooling:

 $v_{\text{growth}} = \mu \left(T_{\text{E}} - T \right)^{\text{n}}$

Macrostructures in Sand Mould Cast Materials

At casting in sand moulds no columnar zone is normally formed usually only equiaxed crystals.

Cast iron is an example of a eutectic alloy cast in a sand mould. Cast iron solidifies into two eutectic structures: *grey cast iron* and *white cast iron*.

Cast iron solidifies by primary precipitation of austenite. When the eutectic point (carbon content 4.3 % C) is reached complex eutectic reactions occur and graphite is precipitated.

The structure formation during the solidification process is reflected distinctly in the appearance of the solidification curve. The structure of cast iron is greatly diversified: four types of grey iron (A, B, C and D-graphite) and white iron:

- coarse flake graphite
- nodular cast iron
- vermicular cast iron
- fine undercooled graphite
- white cast iron, austenite/cementite.

Grey cast iron is easier to machine and has much better mechanical properties than white iron. It is thus desirable that cast iron solidifies as grey cast iron.

White iron formation at solidification of cast iron is favoured by *all* factors, which create high undercooling. Low cooling rate and low growth rate promotes grey iron solidification.





Microsegregation

When an alloy solidifies by dendritic growth the concentration of the alloying element will be distributed unevenly in the material. This phenomenon is called microsegregation.

If the partition constant $k = \frac{x^{s}}{x^{L}} < 1$ the alloying element will concentrate in the remaining melt during the solidification process. If the diffusion rate in the solid phase is low the last solidified parts will have a higher content of the alloying element than the first solidified parts.

If k > 1 the opposite is true.

Τ $T_{\rm M}$ L T° Т х x^{s} $x^{\circ} x^{\perp}$

Sheil's Segregation Equation

Provided that

- the convection and diffusion of the allying element in the melt in each moment is so violent that the melt has an equal composition x^{L} everywhere
- the diffusion of the alloying element in the solid phase is so slow that it can be completely neglected
- a local equilibrium exists at the interface between the solid phase and the melt. The equilibrium can be described by a partition constant

then Scheil's segregation equation is valid:

$$x^{L} = \frac{x^{s}}{k} = x^{o} \cdot (1 - f)^{-(1-k)}$$

Special case:

The fraction, which solidifies with eutectic structure f_{E} , has the concentration of the alloying element equal to

$$x_{\rm E}^{\rm L} = x^{\rm o} \cdot (1 - f_{\rm E})^{-(1-k)}$$





Lever Rule

If the diffusion is rapid in the solid phase then the lever rule is valid.

$$x^{L} = \frac{x^{s}}{k} = \frac{x^{o}}{[1 - f(1 - k)]}$$

Solidification Processes in Alloys

Solidification Process

Alloys have two solidification fronts.

The alloy starts to solidify at the first solidification front. Its temperature is the liquidus temperature $T_{\rm L}$.

The solidification is complete at the second solidification front. Its temperature is the solidus temperature T_s .

Between the two solidification fronts there is a two-phase region with solid phase and melt.



The basis for calculation of temperatures, concentration of alloying element and fraction of solid phase is the heat equation:

$$\left(\rho c_{p} + \rho \cdot \left(-\Delta H\right) \cdot \frac{\mathrm{df}}{\mathrm{d}T}\right) \cdot \frac{\partial T}{\partial t} = k \cdot \frac{\partial^{2} T}{\partial y^{2}}$$

which is integrated after introduction of a convenient expression of df/dT. Two alternatives for df/dT are





If Scheil's equation is valid:

$$\frac{\mathrm{df}}{\mathrm{d}T} = \frac{-1}{1-\mathrm{k}} \cdot (T_{\mathrm{M}} - T)^{-\frac{2-\mathrm{k}}{1-\mathrm{k}}} \cdot (T_{\mathrm{M}} - T^{\circ})^{\frac{1}{1-\mathrm{k}}}$$

If the lever rule is valid:

$$\frac{df}{dT} = \frac{-(T_{\rm M} - T^{\rm o})}{(1 - k) \cdot (T_{\rm M} - T)^{2}}$$

A simple alternative to solving the heat equation is calculation of desired quantities by aid of basic equations of heat transport.

Solidification Interval and Solidification Time of Alloys

Solidification Interval: $\Delta T = T_{\rm L} - T_{\rm s}$

Solidification Time:

In alloys with a large solidification interval the solidification time can be calculated by integration of the differential equation

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = V_{\mathrm{o}}\rho_{\mathrm{metal}} \frac{\mathrm{d}T}{\mathrm{d}t} \cdot \left[c_{\mathrm{p}}^{\mathrm{metal}} + \left(-\Delta H\right) \cdot \frac{\mathrm{d}f}{\mathrm{d}T}\right]$$

If the heat flux is constant during cooling and solidification:

$$c_{\rm p}^{\rm L} \cdot (-\frac{\mathrm{d}T}{\mathrm{d}t}) = (-\Delta H) \cdot \frac{\mathrm{d}f}{\mathrm{d}t}$$

The total solidification time corresponds to f = 1.

$$\theta = \frac{-\Delta H}{c_{\rm p}^{\rm L} \cdot \left(-\frac{\mathrm{d}T}{\mathrm{d}t}\right)}$$

Scheil's Modified Segregation Equation

Back Diffusion

The rapid diffusion in the solid phase eliminates or reduces the concentration gradient of the alloying element. It leads to an exchange of atoms between the liquid and solid phases. The return of alloying atoms from the liquid to the solid is called back diffusion.

Scheil's Modified Segregation Equation

If the back diffusion in the solid phase is considered Scheil's modified equation is valid

$$x^{L} = x^{\circ} \cdot \left(1 - \frac{f}{1 + D^{\circ} \cdot \frac{4\theta}{\lambda^{2}} \cdot k}\right)^{-(l-k)} = x^{\circ} \cdot \left(1 - \frac{f}{1 + B}\right)^{-(l-k)}$$

where $B = \frac{4D^{s}\theta k}{\lambda^{2}}$ is a correction term, which depends on the back diffusion in the solid phase.

- Calculation of the Concentration of the Alloying Element as a Function of the Fraction Solid Phase $f = 2 y/\lambda$
 - At high diffusion rates the lever rule is valid
 - At *low* diffusion rates *Scheil's modified segregation equation* is valid.
 - Sheil's segregation equation (10) on page 8 is valid at solidification processes where more than 10 % of the structure is eutectic and the diffusion rate is low.
 - When $D^{s} = 10^{-11} \text{ m}^{2}/\text{s}$ it is difficult to decide which relation one should use. The cooling rate is allowed to decide.
 - At low cooling rates the lever rule is valid.
 - At *high* cooling rates *Sheil's modified equation* agrees best with reality.



Degree of Microsegregation

$$S = \frac{x_{max}^{s}}{x_{min}^{s}} = \left(\frac{B}{1+B}\right)^{-(1-k)} \text{ where } \qquad B = \frac{4D^{s}\theta k}{\lambda^{2}}$$

- The *higher* the k-value is, the *lower* will the S-value be.
- The *higher* the diffusion rate is the *lower* will the S-value be.
- The *higher* the cooling rate is the *higher* will the S-value be.

Interstitially solved elements diffuse easier through a crystal lattice than substitutionally solved alloying elements, which are parts of the lattice. Thus the former ones have higher diffusion rate and consequently lower segregation degree than the latter ones.

Solidification Processes and Microsegregation in in Iron-Base Alloys

The solidification process in iron-base alloys starts with dendritic solidification and precipitation of either ferrite (δ) or austenite (γ).

The microsegregation in the two cases is completely different. The difference depends on two factors.

- differences in diffusion rate of the alloying element in ferrite respectively austenite.
- differences in partition constants of the alloying element in δ/L respectively γ/L .

The degree of segregation S can be calculated by aid of the phase diagram, the diffusion rate of the alloying elements and the cooling rate.







The solution can in this case be written

$$x = x^{\circ} + \left(x_{\max}^{M} - x^{\circ}\right) \cdot \sin \frac{2\pi y}{\lambda_{den}} \cdot e^{-\frac{4\pi^{2} D}{\lambda_{den}^{2}} \cdot t}$$

and the amplitude of the concentration of the alloying element as a function of time will be

$$\frac{x^{M} - x^{o}}{x_{max}^{M} - x^{o}} = e^{-\frac{4\pi^{2} D}{\lambda_{den}^{2}} t}$$

By aid of these two equations it is possible in simple cases to get an idea of the effect of a heat treatment of the cast material. Alternatively the treatment time *t*, required to get a desired concentration change of the alloying element, can be calculated.

$$t = \frac{\lambda_{\text{den}}^2}{4\pi^2 D} \cdot \ln \frac{x_{\text{max}}^M - x^\circ}{x^M - x^\circ}$$

In order to get a homogenous material in shortest possible time it is important to start with a material, which has *small* dendrite arm distances. Short dendrite arm distances are obtained at high solidification and cooling rates.

It is a significant advantage to cast thin sections by continuous casting rather than to cast big ingots, which later are rolled to thin dimensions.

Dissolution of Secondary Phases

When a binary alloy cools a primary α -phase is precipitated. When *T* has decreased to the eutectic temperature, both an α -phase and a θ -phase, with a high concentration of the alloying element, are precipitated.

The non-desired θ -phase is a secondary phase, which can be dissolved by heat treatment. We will discuss the onedimensional case, i. e. when the secondary phase consists of bands at regular distances from each other.



Case I:

The diffusion fields (the concentration field as a function of distance) from adjacent stripes overlap. The concentration in each point between two stripes is the sum of two terms, caused by the diffusion from each stripe.

Time for complete dissolution of the secondary phase:

/

$$t = -\frac{\lambda_{\text{den}}^{2}}{\pi^{2}D} \cdot \ln\left(1 - \frac{f_{0}^{\theta} \cdot \left(x^{\theta} - x^{\text{m}}\right)}{x^{\alpha/\theta} - x^{\text{m}}}\right)$$

The fraction g of the stripes dissolved during the treatment time t_g :

$$g = \frac{x^{\alpha/\theta} - x^{m}}{f_{o}^{\theta} (x^{\theta} - x^{m})} \cdot \left(1 - e^{-\frac{\pi^{2}Dt}{\lambda_{den}^{2}}}\right)$$

The time required to dissolve the fraction g of the stripes:

$$t_{\rm g} = -\frac{\lambda_{\rm den}^2}{\pi^2 D} \cdot \ln\left(1 - \frac{g f_{\rm o}^{\theta} \cdot \left(x^{\theta} - x^{\rm m}\right)}{x^{\alpha/\theta} - x^{\rm m}}\right)$$

The dissolving time is proportional to the square of the dendrite arm distance. The larger λ_{den} is the *longer* will the dissolution time be.

The higher the treatment temperature is the shorter will the dissolution time be.





Case II:

The diffusion fields (the concentration profile as a function of the distance) of adjacent bands of the secondary phase do *not* overlap.

This is an extreme case, which occurs when the concentration of the alloying element in the matrix is so strongly subsaturated that the diffusion fields do not overlap until the dissolution is finished.

Time for complete dissolution of the secondary phase:

$$t = \frac{b^2}{2D} \cdot \frac{\left(x^{\theta} - x^{m}\right)^2}{\left(x^{\alpha/\theta} - x^{m}\right)^2}$$

The fraction g of the bands dissolved during the treatment time t_g :

$$g = \frac{\sqrt{2D t_g}}{\mathbf{f}_{o}^{\theta} \cdot \lambda_{den}} \cdot \frac{\left(x^{\alpha/\theta} - x^{m}\right)^2}{\left(x^{\theta} - x^{m}\right)^2}$$

The time required to dissolve the fraction g of the bands:

$$t_{g} = \frac{\left(g f_{o}^{\theta} \lambda_{den}\right)^{2}}{2D} \cdot \frac{\left(x^{\theta} - x^{m}\right)^{2}}{\left(x^{\alpha/\theta} - x^{m}\right)^{2}}$$

Change of Casting Structure during Cooling and Plastic Deformation

When a metal is exposed to plastic deformation, the segregation pattern will generally change. At plastic deformation areas with different concentration of alloying elements are stretched into bands in the deformation direction. A lamella structure arises.

In low-alloyed steels this lamella structure may appear as bands of ferrite or perlite.

In a plastically deformed material the mechanical properties of the material will be different in different directions relative to the deformation direction.



Solubility of Gases in Metals

Gas is in most cases more soluble in a metal melt than in the solid phase. When a metal melt solidifies this can result in gas precipitation in the interdendritic areas.

If heat has to be added in order to dissolve the gas in the metal melt (*endothermic reaction*) the solubility curves have a *positive* slope and gas precipitation may occur. This is true for most of the common metals, for example Fe, Al, Cu and Mg.

If the solubility process is *exothermic* the solubility curves have *negative* slopes and gas precipitation does not occur.

Pores

Pores reduce the quality of castings and ingots. Efforts are made by various methods to prevent the formation of gas pores.

Gas Concentrations

The gas concentration in the melt and the *partial* pressure of the gas in the surrounding atmosphere are in equilibrium with each other.

$$\left[\underline{\mathbf{G}}\right] = const \cdot \sqrt{p_{\mathbf{G}_2}}$$

Sievert's law

Even if the gas concentration in the melt initially is low and far from the solubility limit, gas precipitation may occur when the melt has reached a certain solidification fraction.

The relation between gas concentration in the melt, the solidification fraction and the partition coefficient is given by

$$x^{L} = \frac{x^{\circ}}{1 - f(1 - k_{x})}$$
 $c^{L} = \frac{c^{\circ}}{1 - f(1 - k_{c})}$





Methods to Minimise or Eliminate Pipes in Ingot Castings

Feeder

At production of castings efforts are made to control the casting process in such a way that the pipe as a whole is located in a feeder and the casting becomes compact.

The size and position of the feeder is of great importance. Two conditions have to be fulfilled:

- The solidification time of the feeder must be equal to or longer than that of the casting. Then the melt in the feeder compensates the solidification shrinkage of the casting.
- The volume of the feeder must have a size, which at least is big enough to enclose the whole solidification shrinkage inside the feeder.

There are several methods to calculate a suitable feeder volume.

• Solidification Shrinkage is defined as

$$\beta = \frac{\rho_{\rm s} - \rho_{\rm L}}{\rho_{\rm s}} = \frac{V_{\rm L} - V_{\rm s}}{V_{\rm L}}$$

$$\rho_{\rm s}$$
 = density of the solid phase

 $\rho_{\rm L}$ = density of the melt

 $V_{\rm s}$ = volume of the solid phase

 $V_{\rm L}$ = volume of the melt

The amount solidified metal in the feeder is equal to the feeder volume minus the total solidification shrinkage in feeder and casting:

$$V_{\rm sm} = V_{\rm f} - \beta \left(V_{\rm c} + V_{\rm f} \right)$$



Suppose that Chvorinov's rule (pages 8-9) is valid for casting and feeder

$$(1-\beta) \cdot \frac{V_{\rm f}}{V_{\rm c}} = \left(\frac{C_{\rm c}}{C_{\rm f}}\right)^{\frac{1}{2}} \cdot \frac{A_{\rm f}}{A_{\rm c}} + \beta$$

or
$$\left(C_{\rm f}\right)^{\frac{1}{2}} A_{\rm f} = 0$$

$$V_{\rm f} = V_{\rm c} \cdot \frac{\left(\frac{C_{\rm c}}{C_{\rm f}}\right) \cdot \frac{A_{\rm f}}{A_{\rm c}} + \beta}{(1 - \beta)}$$

$$CFR = \frac{t_{\text{total}} - t_{\text{initial}}}{t_{\text{total}}}$$

where

It is possible to use CFR to calculate the size of a feeder. If we assume that the condition $t_c = t_{bscf}$ is valid this relation will be

$$(1-\beta) \cdot \frac{V_{\rm f}}{V_{\rm c}} = \left(\frac{C_{\rm c}}{C_{\rm f}} \cdot \left(1 - \frac{\rm CFR}{100}\right)\right)^{\frac{1}{2}} \cdot \frac{A_{\rm f}}{A_{\rm c}} + \beta$$



Density inversion in the two-phase region is possible in alloy systems if the lighter alloying element (compared to the matrix element) has a normal segregation respectively the heavier alloying element (compared to the matrix element) has an inverted segregation.

Condition for occurrence of freckles:

$$\frac{K_{\rm p} g\beta (T_{\rm L} - T_{\rm S})^4}{v_{\rm kin} D G^3} \ge 1700$$

or alternatively

$$1700 \le \frac{K_{\rm p} g \cdot \Delta \rho \left(T_{\rm L} - T_{\rm S}\right)^3}{\rho v_{\rm kin} D G^3}$$

The casting process must be designed in such a way that freckles are avoided. If a temperature gradient G, which is large enough, is used freckles can not appear.

 $G > G_{\rm cr} \implies$ No freckles



Macrosegregation at Unidirectional Solidification

When an alloy solidifies by horizontal unidirectional solidification the density distribution causes natural convection in the melt and the two-phase region, which leads to macrosegregation in the material.

Within the two-phase region the density of the melt can be either higher or lower than the density of the melt outside.

The gravity constant influences the size of the macrosegregation strongly. Space experiments show that a decreased gravitation force reduces the convection strongly and macrosegregations of this type can be avoided.

Macrosegregation in Ingots

When crystals in an ingot sediments there is an exchange of interdendritic melt included inside the crystals and the surrounding bulk melt. The exchanged interdendritic melt remains in the upper parts of the ingot and crystals with partly more pure melt sinks to the bottom. The result is positive segregation in the upper parts of the ingot and negative segregation in the lower parts.



Sedimentation Segregations

Sedimentation of uniaxed crystals results in an uneven distribution of the alloying elements. Each crystal carries a certain amount of the melt when it sediments.

The average concentration of the alloying element in the sediment zone:

$$c^{+} = \frac{c^{\mathrm{s}} + \mathrm{W}c^{\mathrm{L}}}{1 + \mathrm{W}}$$

