Analogous modifications have to be made in *all other* equations, where the heat of solution and the heat capacitivity are involved, for example in equations (46) and (48) on page 34. However, this is only necessary in some cases.

In the rest of this chapter we will assume that the heat of fusion and the heat capacitivity are constants. In chapter 5, when we discuss rapid solidification processes, it will be necessary to consider the effect of the vacancies.

4.4 Heat Transport at Component Casting

4.4.1 Temperature Distribution at Sand Mould Casting

In the preceding section we have treated the temperature distribution in mould and metal with ideal cooling. In reality the contact between melt and mould is very seldom ideal.

At casting in sand moulds the solution in section 3.4.2 of the general law of heat conduction, which presumes good heat contact between mould and melt, can be applied even if this condition is not fulfilled. The reason is the poor thermal conductivity of sand. Below we will discuss how the temperature T_i at the interface between mould and metal varies as a function of the properties of the mould material.

Calculation of the Temperature T_i between Sand Mould and Metal

The contact between metal and sand mould is not ideal. Due to the poor heat conductivity of the sand mould (of the magnitude 1 W/mK) it is reasonable, however, to assume that the temperature distribution is about the same as if the cooling were ideal.

In section 4.3 we have calculated the temperature distribution in mould and metal at casting with ideal heat contact between mould and melt. The result of these calculations is summarised below.

$$T_{\text{mould}} = A_{\text{mould}} + B_{\text{mould}} \cdot \text{erf}\left(\frac{y}{\sqrt{4\alpha_{\text{mould}} t}}\right) \qquad (19_{\text{mould}})$$

$$T_{\text{metal}} = A_{\text{metal}} + B_{\text{metal}} \cdot \text{erf}\left(\frac{y}{\sqrt{4\alpha_{\text{metal}} t}}\right) \qquad (19_{\text{metal}})$$

where A_{mould}, B_{mould}, A_{metal} and B_{metal} are constants, the values of which depend of the material properties of the mould material and the melt. We found the four following relations:

$$A_{\text{mould}} = A_{\text{metal}} = T_{\text{o}} + \frac{T_{\text{L}} - T_{\text{o}}}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} + \text{erf}\,\lambda} \cdot \frac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} \qquad (35)$$

$$B_{\text{metal}} = \frac{T_{\text{L}} - T_{\text{o}}}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} + \text{erf}\,\lambda} \qquad (33)$$

$$B_{\text{mould}} = \frac{T_{\text{L}} - T_{\text{o}}}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} + \text{erf}\,\lambda} \cdot \frac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} \qquad (34)$$

where T_{o} is the temperature of the surroundings. The constants in equations (33), (34) and (35) contain a common constant λ, which is determined from equation (36) on page 28

$$\frac{c_{\text{p}}^{\text{metal}}(T_{\text{L}} - T_{\text{o}})}{-\Delta H} = \sqrt{\pi} \cdot \lambda\, \text{e}^{\lambda^2} \cdot \left(\sqrt{\frac{k_{\text{metal}}\rho_{\text{metal}}\, c_{\text{p}}^{\text{metal}}}{k_{\text{mould}}\rho_{\text{mould}}\, c_{\text{p}}^{\text{mould}}}} + \text{erf}\,\lambda\right) \qquad (36)$$

These equations will be applied on the present case. At the interface between the sand mould and metal we have

$$T_{\text{i}} = A_{\text{mould}} = A_{\text{metal}} \qquad (57)$$

where T_{i} is the temperature at the interface. These relations are found by replacing y by zero in equations (19_{mould}) and (19_{metal}).

Equation (35) can be transformed into

$$A_{\text{mould}} = A_{\text{metal}} = T_{\text{i}} = T_{\text{o}} + \frac{T_{\text{L}} - T_{\text{o}}}{1 + \dfrac{\text{erf}\,\lambda}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}}}} \tag{58}$$

Sand has poor thermal conductivity compared to metals, which means that $k_{\text{metal}} >> k_{\text{mould}}$.

Because $\alpha = \dfrac{k}{\rho c_{\text{p}}}$ (equation (11) on page 8) we get

$$\frac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} = \frac{k_{\text{metal}}\sqrt{\dfrac{k_{\text{mould}}}{\rho_{\text{mould}}\, c_{\text{p}}^{\text{mould}}}}}{k_{\text{mould}}\sqrt{\dfrac{k_{\text{metal}}}{\rho_{\text{metal}}\, c_{\text{p}}^{\text{metal}}}}} = \frac{\sqrt{k_{\text{metal}}\,\rho_{\text{metal}}\, c_{\text{p}}^{\text{metal}}}}{\sqrt{k_{\text{mould}}\,\rho_{\text{mould}}\, c_{\text{p}}^{\text{mould}}}} >> 1 \tag{59}$$

The value of erf λ in equation (58) lies between 0 and 1. The second term in the denominator in equation (58) will thus be very small and the denominator ≈ 1. In this case we get

$$T_{\text{i}} = T_{\text{i metal}} = T_{\text{i mould}} \approx T_{\text{L}} \tag{60}$$

- At casting in a sand mould the temperature at the interface between the metal and the mould is approximately equal to the liquidus temperature of the melt.

4.4.2 Solidification Rate and Solidification Time at Sand Mould Casting. Chvorinov´s Rule

Dry Sand Mould

The result above that $T_{\text{i}} = T_{\text{i metal}} = T_{\text{i mould}} \approx T_{\text{L}}$ is *very* important and is frequently used at casting in sand moulds.

The solidification process occurs mainly in the way illustrated in figure 10 on page 15. The temperature distribution in the sand mould and in the metal is illustrated in figure 20.

Because sand has a poor thermal conductivity the heat transport through the sand mould is the ”bottle neck”. The solidification process is completely controlled by the heat conduction through the sand mould.

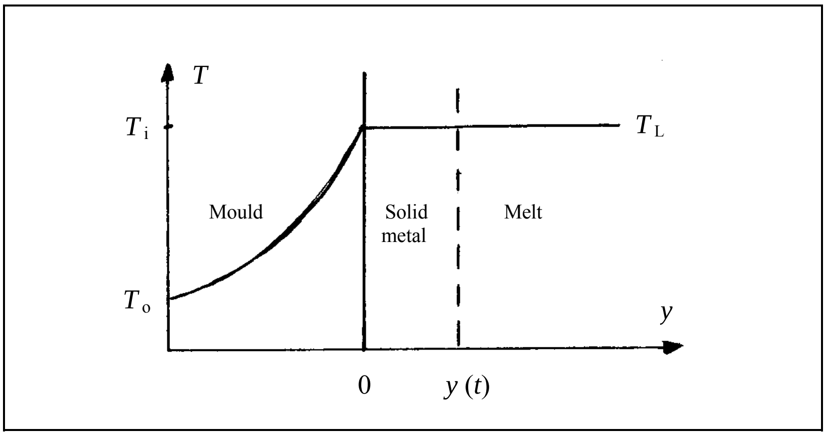

Figure 20.
Sketch of the temperature profile at casting in a sand mould.

We start with the following assumptions:

1. The conductivity of the metal is very large compared to that of the sand mould.
2. During the casting the temperature of the mould wall immediately becomes equal to the temperature T_L of the melt and keeps this temperature throughout the whole solidification process.
3. At large distances from the interface the temperature of the mould is equal to the room temperature T_o.

The temperature distribution in the metal is simple in this case and given by

$$T_{\text{metal}} = T_L \tag{61}$$

It is not necessary to solve any equation in this case to find the temperature in the metal unless the thickness is very large and/or the mould is comparatively small. Instead we can restrict the calculations to the heat conduction in the mould.

The solution to the general law of heat conduction

$$\frac{\partial T_{\text{mould}}}{\partial t} = \alpha_{\text{mould}} \frac{\partial^2 T_{\text{mould}}}{\partial y^2} \tag{62}$$

can, with use of customary designations, be written:

$$T_{\text{mould}} = A_{\text{mould}} + B_{\text{mould}} \cdot \text{erf}\left(\frac{y}{\sqrt{4\alpha_{\text{mould}}\, t}}\right) \tag{63}$$

In order to determine the constants A_{mould} and B_{mould} we will use known boundary conditions, which have to be fulfilled.

Boundary Condition 1:

At the interface $y = 0$ $T(0, t) = T_{\text{i}}$

Boundary Condition 2:

In the mould $y = -\infty$ $T(-\infty, t) = T_{\text{o}}$

These pairs of values are inserted into equation (63):

$$T_{\text{i}} = A_{\text{mould}} + B_{\text{mould}} \cdot \operatorname{erf}(0) = A_{\text{mould}} \tag{64}$$

$$T_{\text{o}} = A_{\text{mould}} + B_{\text{mould}} \cdot \operatorname{erf}(-\infty) = A_{\text{mould}} - B_{\text{mould}} \tag{65}$$

We can solve A_{mould} and B_{mould} from this equation system:

$$\begin{aligned} A_{\text{mould}} &= T_{\text{i}} \\ B_{\text{mould}} &= T_{\text{i}} - T_{\text{o}} \end{aligned} \tag{66}$$

If we insert these values into equation (63) we get the solution to the general law of heat conduction, i. e. the temperature distribution in the mould as a function of position and time

$$T_{\text{mould}}(y,t) = T_{\text{i}} - \left(T_{\text{i}} - T_{\text{o}}\right) \cdot \operatorname{erf}\left(\frac{y}{\sqrt{4\alpha_{\text{mould}}\, t}}\right) \tag{67}$$

The amount of heat per unit area, which is transported into the mould per unit time through the interface between the mould and the metal, is given by the expression

$$\frac{\partial q(0,t)}{\partial t} = k_{\text{mould}} \cdot \frac{\partial T(0,t)}{\partial y} \tag{68}$$

Equation (67) is derived with respect to y and the derivative is inserted into equation (68):

$$\frac{\partial q}{\partial t} = k_{\text{mould}} \cdot \left(T_{\text{i}} - T_{\text{o}}\right) \cdot \frac{2}{\sqrt{\pi}} \cdot \mathrm{e}^{-\frac{y^2}{4\alpha_{\text{mould}}\, t}} \cdot \frac{1}{\sqrt{4\alpha_{\text{mould}}\, t}}$$

If we insert $y = 0$ the exponential factor becomes equal to 1 and we get

$$\frac{\partial q}{\partial t} = k_{\text{mould}} \cdot \left(T_{\text{i}} - T_{\text{o}}\right) \cdot \frac{1}{\sqrt{\pi \alpha_{\text{mould}}\, t}} \tag{69}$$

The relation $\alpha_{\text{mould}} = \dfrac{k_{\text{mould}}}{\rho_{\text{mould}} \cdot c_{\text{p}}^{\text{mould}}}$ is inserted into equation (69) and we get

$$\frac{\partial q}{\partial t} = \sqrt{\frac{k_{\text{mould}}\, \rho_{\text{mould}}\, c_{\text{p}}^{\text{mould}}}{\pi t}} \cdot (T_{\text{i}} - T_{\text{o}}) \tag{70}$$

The amount of heat, which passes the interface per unit area into the mould, consists only of solidification heat because the temperature (T_{L}) is constant in the solid phase and the melt. The solidification heat per unit time will be

$$\frac{\partial q}{\partial t} = \rho_{\text{metal}} \cdot (-\Delta H) \cdot \frac{\mathrm{d}y_{\text{L}}}{\mathrm{d}t} \tag{71}$$

$\mathrm{d}y_{\text{L}}/\mathrm{d}t$ is the solidification rate, i. e. the velocity of the solidification front in the melt. We can get the thickness of the solidifying layer as a function of time by inserting the expression for the partial time derivative of q (equation (70)) into equation (71), solve the solidification rate and integrate

$$y_{\text{L}}(t) = \int_0^t \frac{T_{\text{i}} - T_{\text{o}}}{\rho_{\text{metal}} (-\Delta H)} \cdot \sqrt{\frac{k_{\text{mould}}\, \rho_{\text{mould}}\, c_{\text{p}}^{\text{mould}}}{\pi}} \cdot \frac{\mathrm{d}t}{\sqrt{t}}$$

or

$$y_{\text{L}}(t) = \frac{2}{\sqrt{\pi}} \cdot \frac{T_{\text{i}} - T_{\text{o}}}{\rho_{\text{metal}} (-\Delta H)} \cdot \sqrt{k_{\text{mould}}\, \rho_{\text{mould}}\, c_{\text{p}}^{\text{mould}}} \cdot \sqrt{t} \tag{72}$$

It is important to notice that the *second* factor contains data for the *metal* and the *third* factor data for the *mould*.

It is seen from equation (72) that

- The thickness of the solidifying layer is a parabolic function of time.

The solidification rate is rapid at the beginning of the solidification process but decreases successively when the solidified layer gets thicker.

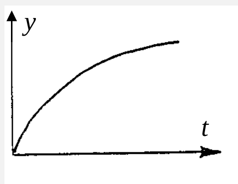

Figure 21.
The solidifying thickness y of a solidifying shell as a function of time at casting in sand moulds.

The geometrical shape of the mould wall also influences its capacity to absorb heat. The heat is transferred more rapidly in a concave mould area than in a planar one because the travel distance is shorter there. The opposite is valid for a convex surface. Heat is transferred more slowly into a convex mould/casting area than in a planar one because the travel distance is longer there. The differences are rather small for simpler moulds, though. Two examples will be discussed below.

Thermal Conduction at Sharp Corners

Cast components often have very irregular shapes and more or less sharp corners.

Figure 22 a shows the isotherms in a mould in the neighbourhood of and at a convex corner, seen from the melt (outer corner). The heat from the walls of the casting can only be transported in the direction of the temperature gradient, i. e. perpendicular to the isotherms. The heat transport per unit area and unit time is proportional to the temperature gradient. The

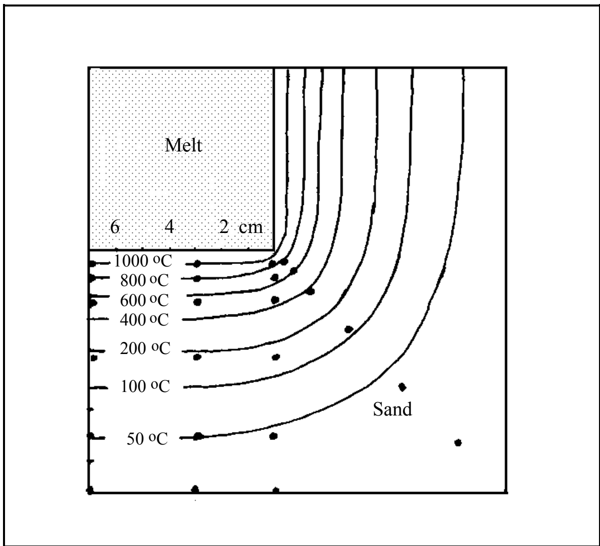

Figure 22 a.
Isotherms in the sand mould outside an outer corner of a casting 15 minutes after the casting process. The cast metal is cast iron. The temperature of the inner surface is 1083 °C.
The points in the figure represent thermoelements. The length scale in the figure corresponds to the distances from the corner.

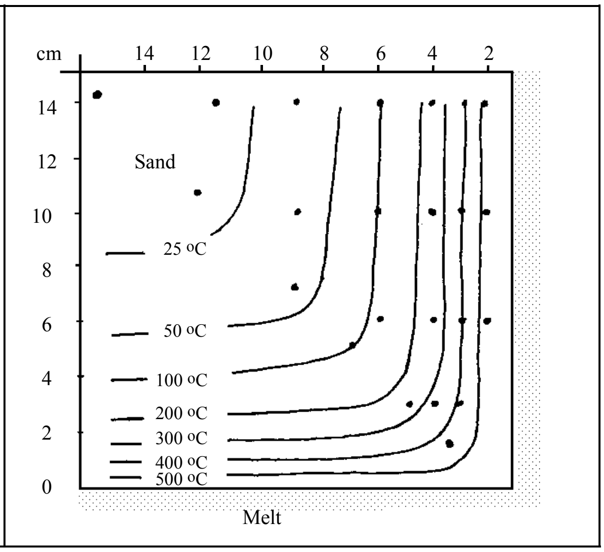

Figure 22 b.
Isotherms in the sand mould inside an inner corner of a casting 15 minutes after the casting process. The cast metal is an aluminium base alloy. The temperature of the inner surface is 548 °C.
The points in the figure represent thermoelements. The length scale in the figure corresponds to the distances from the corner.

closer the isotherms are situated, the larger is the temperature gradient. It can be seen from figure 22 a that the heat conduction is larger at an outer corner in the direction of the diagonal than at a planar surface.

Figure 22 b shows the isotherms in a mould in the neighbourhood of and at a concave corner, seen from the melt (inner corner). The larger the distances between the isotherms are the smaller is the temperature gradient and the smaller is the heat flux from the melt. Figure 22 b shows that the distances between the isotherms in the direction of the diagonal are larger than the ones, perpendicular to a planar surface. The heat conduction is thus smaller at an inner corner in the direction of the diagonal than that at a planar surface.

Chvorinov´s Rule

A useful approximation is to assume that every unit area of the mould wall has a constant ability to absorb heat independent of its geometrical shape and its position on the surface of the casting. If we make this assumption we can replace $y_L(t)$ in equation (72) with V_{metal}/A in

$$\frac{V_{metal}}{A} = \frac{2}{\sqrt{\pi}} \cdot \frac{T_i - T_o}{\rho_{metal}(-\Delta H)} \cdot \sqrt{k_{mould}\rho_{mould}c_p^{mould}} \cdot \sqrt{t_{total}} \qquad (73)$$

where

V_{metal} = total volume of the solidified casting

A = total area of the interface between the mould and the metal

t_{total} = the total solidification time.

Equation (73) can be written in a simpler way as

$$t_{total} = C \cdot \left(\frac{V_{metal}}{A}\right)^2 \qquad \text{Chvorinov´s rule} \qquad (74)$$

- The total solidification time is proportional to the square of the volume of the casting and inversely proportional to the square of the contact area between the sand mould and the casting.

The constant C is obtained in terms of material constants by identification of equations (73) and (74) (see page 50).

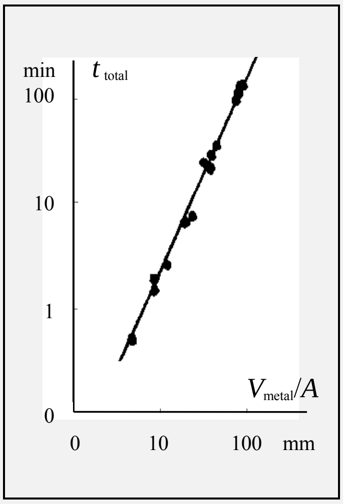

Figure 23.
Chvorinov´s experimental results of the solidification time of castings as a function of their volume/area ratio. The scale is logarithmic on both axes and the slope of the line is 2.

Equation (74) is very well known and extraordinary useful for relative comparisons of the solidification times of castings, made of the same material.

Constant in Chvorinov´s rule:

$$C = \frac{\pi}{4} \cdot \frac{\rho_{\text{metal}}^2 \left(-\Delta H\right)^2}{(T_i - T_o)^2 k_{\text{mould}} \rho_{\text{mould}} c_p^{\text{mould}}}$$

Chvorinov has verified his rule by experiments on sand mould castings of varying shapes and sizes, from 10-mm castings to 65-ton ingots. Later it has been shown that a similar rule is valid for other castings as well, such as some (but not all) ingot castings. This matter will be further discussed on page 9 in chapter 5.

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Example 4.

In a foundry thin castings are produced. The thickness of the cast iron is just thick enough to prevent white solidification. A new mould production method is introduced, which gives denser packing of the sand in the mould. This increases the risk of white solidifying, which gives poor quality of the casting. The density of the mould material increases with 20 % and the conductivity of the sand increases with 10 %.

The casting has earlier been done in such a way that the temperature of the melt in the mould at the end of the casting process has been exactly equal to the solidus temperature. By how many degrees must the casting temperature be increased if you tolerate the same maximum risk of white solidification as before?

$-\Delta H = 170$ kJ/kg and $c_p^{\text{metal}} = 0.42$ kJ/kgK.

Solution:

We assume that the casting temperature has to be increased by the amount ΔT. The heat capacitivity c_p^{mould} will not change be-cause of the new casting method.

We apply Chvorinov´s rule to calculate the solidification time at casting according to the old method (index 1) and the new method (index 2).

$$t_1 = \left(\frac{\sqrt{\pi}}{2} \cdot \frac{\rho_{\text{metal}}(-\Delta H)}{T_{\text{i}} - T_{\text{o}}} \cdot \frac{1}{\sqrt{k_{\text{mould 1}} \ \rho_{\text{mould 1}} \ c_{\text{p}}^{\text{mould 1}}}} \right)^2 \cdot \left(\frac{V_{\text{metal}}}{A} \right)^2$$

$$t_2 = \left(\frac{\sqrt{\pi}}{2} \cdot \frac{\rho_{\text{metal}}(-\Delta H + c_{\text{p}}^{\text{metal}} \Delta T)}{T_{\text{i}} - T_{\text{o}}} \cdot \frac{1}{\sqrt{k_{\text{mould 2}} \ \rho_{\text{mould 2}} \ c_{\text{p}}^{\text{mould 2}}}} \right)^2 \cdot \left(\frac{V_{\text{metal}}}{A} \right)^2$$

The solidification time must be the same in both cases, i.e. $t_1 = t_2$, to prevent an increased risk of white solidification. This gives the equality

$$\frac{-\Delta H}{\sqrt{k_{\text{mould 1}} \ \rho_{\text{mould 1}}}} = \frac{-\Delta H + c_{\text{p}}^{\text{metal}} \Delta T}{\sqrt{k_{\text{mould 2}} \ \rho_{\text{mould 2}}}} \qquad (3')$$

or

$$\frac{-\Delta H + c_{\text{p}}^{\text{metal}} \Delta T}{-\Delta H} = \frac{\sqrt{k_{\text{mould 2}} \ \rho_{\text{mould 2}}}}{\sqrt{k_{\text{mould 1}} \ \rho_{\text{mould 1}}}} = \sqrt{1.1 \times 1.2} \qquad (4')$$

The values given in the text are inserted and we get $\Delta T = 60\ ^{\text{o}}\text{C}$

Answer: The casting temperature increase should be 60 $^{\text{o}}$C.

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Relation between Solidification Time and the Ratio V/A for Spherical and Cylindrical Moulds

For spherical and cylindrical moulds it is possible to derive a more exact expression than Chvorinov´s law, which gives a relation between the casting time and the ratio *V/A*. In these cases the partial differential equation for heat conduction can be written

$$\frac{\partial T}{\partial t} = \alpha_{\text{mould}} \left(\frac{\partial^2 T}{\partial r^2} + \frac{\text{n}}{r} \cdot \frac{\partial T}{\partial r} \right) \qquad (75)$$

where

r = the radius of the casting
n = 1 for a cylinder
n = 2 for a sphere.

If a similar derivation is performed like the one which led to equation (73), the result will be

$$\frac{V_{\text{metal}}}{A} = \cdot \frac{T_{\text{i}} - T_{\text{o}}}{\rho_{\text{metal}}(-\Delta H)} \cdot \left(\frac{2}{\sqrt{\pi}} \cdot \sqrt{k_{\text{mould}} \rho_{\text{mould}} c_{\text{p}}^{\text{mould}}} \cdot \sqrt{t_{\text{total}}} + \frac{\text{n}k_{\text{mould}} t_{\text{total}}}{2r} \right) \tag{76}$$

A comparison between equations (76) and (74) shows that the more k_{mould} decreases and r increases, the better will Chvorinov´s simple approximation be valid. There is better agreement for a cylinder than for a sphere. For a given *V/A* ratio a sphere solidifies more rapidly than a cylinder, which solidifies more rapidly than a plate.

Example 5.

Determine the solidification time for a steel cylinder with a diameter of 15 cm, which is cast in a sand mould. The height of the cylinder is much larger than its diameter.

The sand mould respectively the steel has the following material constants:

The thermal conductivity of the sand	$k_{\text{mould}} = 0.63$ J/mKs
The density of the sand	$\rho_{\text{mould}} = 1.61 \cdot 10^3$ kg/m^3
The heat capacitivity of the sand	$c_{\text{p}}^{\text{mould}} = 1.05 \cdot 10^3$ J/kgK
Solidification temperature of the steel	$T_{\text{L}} = T_{\text{i}} = 90$ $^{\text{o}}$C
	$T_{\text{o}} = 23$ $^{\text{o}}$C
Solidification heat of the steel	$-\Delta H = 272$ kJ/kg.

Solution:

We apply equation (76)

$$\frac{V_{\text{metal}}}{A} = \cdot \frac{T_{\text{i}} - T_{\text{o}}}{\rho_{\text{metal}}(-\Delta H)} \cdot \left(\frac{2}{\sqrt{\pi}} \cdot \sqrt{k_{\text{mould}} \rho_{\text{mould}} c_{\text{p}}^{\text{mould}}} \cdot \sqrt{t_{\text{total}}} + \frac{\text{n}k_{\text{mould}} t_{\text{total}}}{2r} \right)$$

and introduce the given values and the height *L* of the cylinder:

$$\frac{V_{\text{metal}}}{A} = \frac{\pi r^2 L}{2\pi rL} = \frac{r}{2} = \frac{7.5}{2} = 3.75 \cdot 10^{-2} \text{ m}$$

$$\frac{T_\mathrm{i} - T_\mathrm{o}}{\rho_\mathrm{metal}(-\Delta H)} = \frac{1490 - 23}{7.8 \cdot 10^3 \cdot 272 \cdot 10^3} = 0.691 \cdot 10^{-6}\ \mathrm{Km^3/J}$$

$$\frac{\mathrm{n}k_\mathrm{mould}}{2r} = \frac{1 \cdot 0.63}{2 \cdot 0.075} = 4.2\ \mathrm{J/m^2sK}$$

A dimension check shows that the dimensions agree. We introduce the calculated and known values into equation (1′):

$$3.75 \cdot 10^{-2} = 0.691 \cdot 10^{-6} \cdot \left(1.16 \cdot 10^3 \cdot \sqrt{t_\mathrm{total}} + 4.2 \cdot t_\mathrm{total}\right) \qquad (2')$$

This is a second order equation of $\sqrt{t_\mathrm{total}}$ which can be written

$$t_\mathrm{total} + 276 \cdot \sqrt{t_\mathrm{total}} - 12921 = 0 \qquad (3')$$

This equation has the roots

$$\sqrt{t_\mathrm{total}} = -138 \pm \sqrt{138^2 + 12921} \qquad (4')$$

The solidification time is associated with the positive root

$$\sqrt{t_\mathrm{total}} = -138 + 179 = 41 \qquad t_\mathrm{total} = 1681\ \mathrm{s} = 28\ \mathrm{min}$$

Answer: The steel cylinder solidifies completely after approximately 28 minutes.

Temperature Distribution in a Moist Sand Mould. Velocity of the Evaporation Front/WaterFront

Sand moulds often contain water, which is evaporated when the sand mould is heated by the melt. This evaporation contributes to the cooling due to the high evaporation heat of water and influences the solidification process for this reason.

The water vapour will condense when it reaches the level in the mould where the temperature is below 100 °C. A waterfront, which moves inside the mould, is formed. Figure 24 describes graphically the temperature distribution in the mould in this case. The evaporation front and the waterfront are identical.

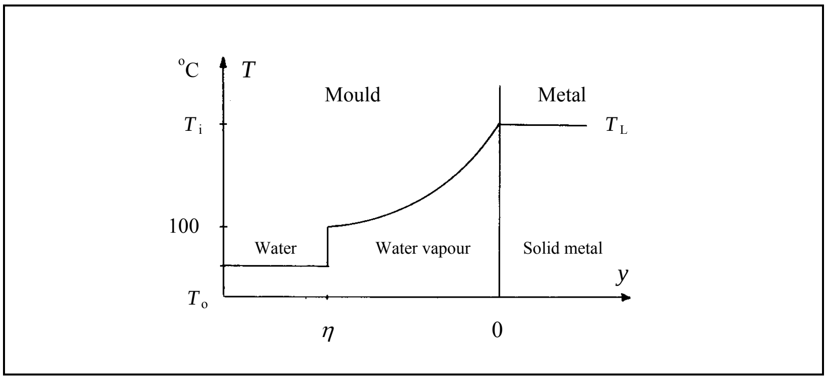


Figure 24.
Temperature distribution in a moist sand mould.

η = the co-ordinate of the waterfront.

Mathematically the temperature distribution can, even in this case, be described by a solution to the law of thermal conduction of the type (for simplicity the index mould has been dropped)

$$T = A + B\,\mathrm{erf}\left(\frac{y}{\sqrt{4\alpha\, t}}\right) \tag{77}$$

Boundary Conditions:

1. At $y = 0$ the temperature at the interface between mould and metal will be

$$T = T_i \tag{78}$$

2. At the waterfront $y = \eta$ (η is always negative, see figure 24) the temperature will be

$$T = 100\ ^{\mathrm{o}}\mathrm{C} \tag{79}$$

The first boundary condition gives the constant $A = T_i$. The second boundary condition permits calculation of the constant *B*.

$$100 = A + B \cdot \mathrm{erf}\left(\frac{\eta}{\sqrt{4\alpha\, t}}\right) \tag{80}$$

For the same reasons as before (page 26) the variable of the function must be constant, i. e.

$$\eta = -\lambda \cdot \sqrt{4\alpha\, t} \qquad (81)$$

where λ is a positive constant, which is determined (compare equation (36) on page 28) by the equation

$$\sqrt{\pi} \cdot \lambda\, e^{\lambda^2} \cdot \operatorname{erf} \lambda = \frac{c_p \cdot (T_i - 100)}{(-\Delta H^{\text{vapour}}) \cdot (\text{fraction of moisture})} \qquad (82)$$

where the fraction of moisture is the weight per cent water based on the weight of the mould. The values of the constants becomes

$$A = T_i \quad \text{and} \quad B = \frac{T_i - 100}{\operatorname{erf} \lambda}$$

λ can be determined by iteration or graphically by aid of figures 14 and 15 on page 29.

λ determines the velocity of the water front in the mould. We get an expression of this velocity by deriving equation (81) with respect to t (compare equation (27) on page 26).

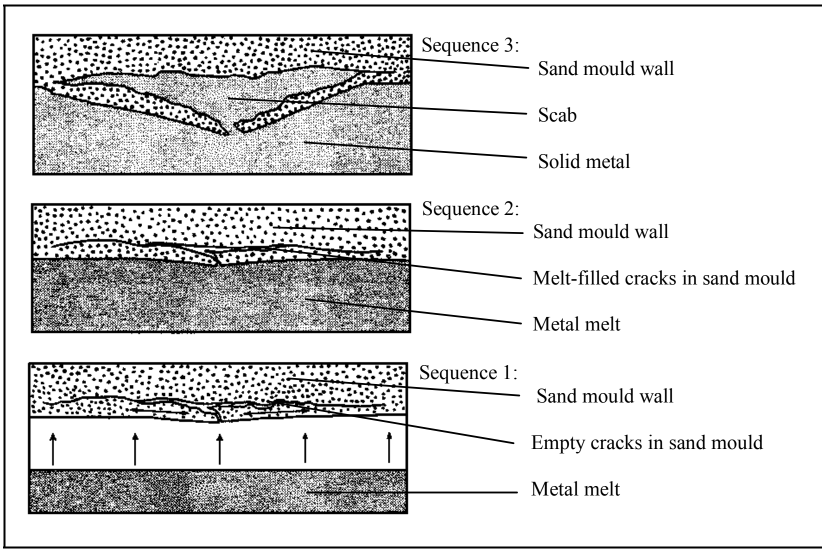
$$\frac{d\eta}{dt} = -\ \lambda \cdot \sqrt{\frac{\alpha}{t}} \qquad (83)$$

where $d\eta/dt$ is the velocity of the water front. $d\eta/dt$ is negative for all values of the time t. The position of the water front is defined by equation (81). It moves in the direction of the negative y-direction with a velocity, defined by equation (83). Equations (81) and (83) are analogous with equations (26) and (27) on page 26.

As mentioned on page 53 a waterfront, which moves forward, is formed in moist sand moulds. The amount of water at the water-front increases successively, which has a negative effect on the mechanical properties of the mould. Figures 25 a and b give two examples of casting faults, which are formed at the formation of a waterfront.

Figure 25 a shows a case where the upper sand mould wall is heated by the melt during the mould filling process and a waterfront is formed in the wall. The moist sand in the mould gets very bad mechanical properties in the layer, which contains the waterfront. When the sand in this layer expands during heating thermal stresses force the sand layer to split up (sequence 1). Later the cracks in the sand get filled with melt (sequence 2) and get enlarged. The appearance of the rough surface after solidification (sequence 3) associates to scab.

Figure 25 b shows a similar case. Here the sand mould gets heated inhomogeneously when a melt stream flows along the lower surface of the mould during the filling process. The bad mechanical properties of the waterfront layer result in this case



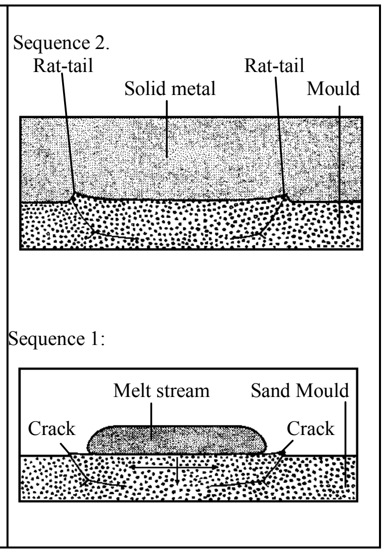


Figure 25 a.
The melt creeps into, enlarges and fills the cracks in the sand mould. A so-called *scab* is formed.

Figure 25 b.
The melt cannot creep into very narrow cracks. Instead so-called *rat-tails* are formed.

in narrow cracks (sequence 1). A mirror pattern of the thin cracks is formed in the casting. The pattern reminds of rat-tails (sequence 2).

4.4.3 Heat Transport at Permanent Mould Casting

The most important permanent mould casting processes are squeeze casting, pressure die casting and gravity die casting, where the metal is poured into the mould. All these methods have been described in chapter 1.

Permanent mould casting is used for a variety of cast components. The dominating cast metal is aluminium.

A sand mould can only be used once, while permanent moulds, made of metal, can be used thousands or millions of times depending on the cast metal. Such moulds have 30-50 times better heat conductivity than sand moulds, which allow high cooling rates and rapid cooling. They are thus most useful for thin component casting.

Heat Transfer at the Interface Mould/Metal

The heat transfer in permanent mould casting mainly depends on the interface between the mould and the cast metal. The heat flux from the hot casting to the cooler mould varies with time and temperature. It depends on

- air gap between mould and casting
- surface roughness and interfacial air or gas films.
- mould coating

The air gap is caused by solidification shrinkage and thermal contraction during cooling of the casting. It has been discussed in section 4.3 and will be further discussed in connection with continuous casting and in chapter 10 where we discuss thermal stresses.

The *surface roughness* of both the mould and the solidifying metal causes a very uneven contact between the two surfaces. The voids are filled with air or some other gas in special cases. Both these effects contribute to the thermal resistance of the interface.

The mould is often covered with a thin film of some ceramic powder, held together by a binder, usually water glass (silicate of sodium). The coating in an aqueous dispersion is normally sprayed on the interior surface of the hot mould (~200 ^{o}C). When the dispersion hits the mould surface the water evaporates immediately and leaves big voids inside the coating.

The *mould coating* has two functions.

1. It protects the mould surface and prolongs its endurance.
2. It prevents the melt from solidifying too early and helps to fill the mould completely before solidification.

The coating layer has a low thermal conductivity, mainly due to about 50 % voids, and forms a thin insulating layer.

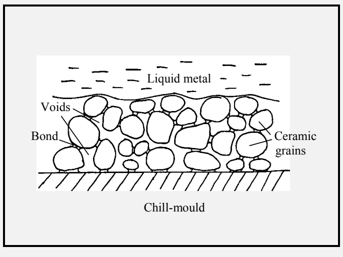


Figure 26.
Ceramic mould coating. The thickness of the layer is normally of the magnitude 1-0.1 mm.

4.4.4 Water-Cooling at Die Casting

Several measures have to be taken to design a die casting equipment in an optimal way. Vents and draw pockets are necessary to prevent air inclusions at the filling of the mould, for example.

The irregular shapes of many castings require special precautions to maintain the temperature at an even and optimal level in all parts of the casting. Some sections, for example gating points where the speed of the injected metal raises the temperature, must be cooled with water to keep the correct temperature.

The water runs in special water tubes, which are drilled in the die block. In some cases they are drilled through the whole block. In other cases the cooling is selective. The water may enter through one short pipe, circulate around the regions to be cooled and leave through another short pipe or nipple.

The design of proper water-cooling will be extensively discussed in section 5.4 in chapter 5. It is a common matter for all casting methods with water-cooling.

4.4.5 Nussel´s Number. Temperature Profile of Mould and Metal at Low Values of Nussel´s Number

On page 32 we found that the heat transfer across the interface metal/mould drastically decreases when the solid metal looses the contact with the mould wall. The solidification process was analysed, which among other things, resulted in an expression for the temperature of the metal at the interface as function of *h*, *k* and the distance y_L of the solidification front from the interface. The expression is equation (45) on page 34:

$$T_{\mathrm{i\,metal}} = \frac{T_L - T_o}{1 + \frac{h}{k} y_L(t)} + T_o \tag{45}$$

We will analyse this relation more closely here. If the heat transfer at the interface between metal and mould is very slow (*h* very small) and/or the thermal conductivity of the solid cast metal is large (k very large) then the second term in the denominator will be small.

At complete solidification y_L has reached its maximum value, which we will call s. At constant values of the temperatures of the melt and the surroundings in equation (45) the temperature profile is obviously determined by the so-called Nussel´s number. It is defined by aid of the relation

$$\mathrm{Nu} = \frac{hs}{k} \tag{84}$$

where

h = heat transfer number for the interface between the mould and the metal
s = value of y_L at complete solidification
k = thermal conductivity of the metal.

Nussel´s number is frequently used as a criterion on the choice of temperature distribution model. If Nu << 1 the simple temperature distribution, illustrated in figure 27, is valid and can safely be used.

If Nu << 1 equation (48) on page 34 can be simplified to

$$t = \frac{\rho(-\Delta H)}{T_L - T_o} \cdot \frac{y_L}{h} \tag{85}$$

The total solidification time is obtained if y_L in equation (85) is replaced by the thickness of the casting at unidirectional cooling. Equation (85) can also be applied on bilateral cooling of castings. In this case s means *half* the thickness of the casting.

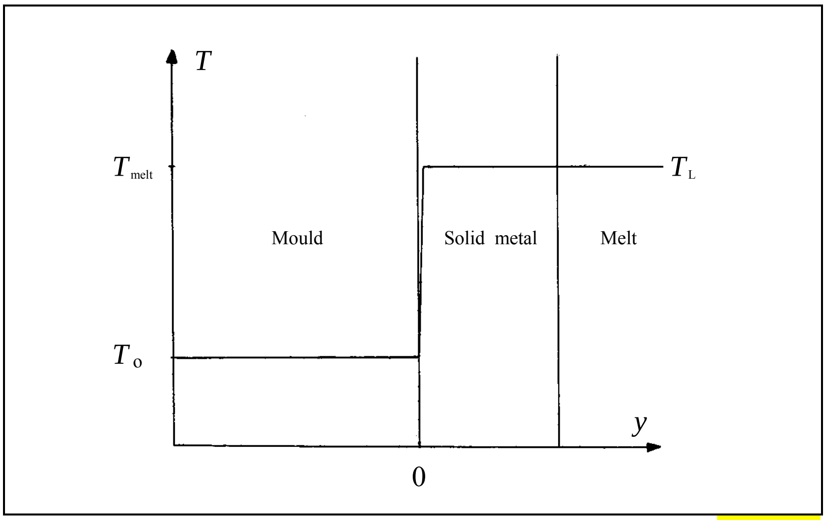


Figure 27.
Temperature distrib
mould and metal fo
lues of Nussel´s nu

The solidification process described above is valid for thin component casting in permanent moulds, i. e. when Nu << 1.

Example 6.

You will produce a thin article for the car industry by pressure die casting. To make the production cheap the article should be cast as rapidly as possible. You can either cast the article in a magnesium alloy or an aluminium alloy. Which one should you use? The heat transfer constant can be regarded as equal in both cases.

Solution:

In order to apply equation (85) and calculate the solidification time in the two cases we have to use the material constants for Mg and Al, taken from tables:

$-\Delta H_{Al} = 3.54 \cdot 10^3 \text{ J/kg}$ $\qquad -\Delta H_{Mg} = 208 \cdot 10^3 \text{ J/kg}$

$\rho_{Al} = 2.69 \cdot 10^3 \text{ kg/m}^3$ $\qquad \rho_{Mg} = 1.74 \cdot 10^3 \text{ kg/m}^3$

Field Code Changed

Field Code Changed

The temperature of the melt is approximately equal to the melting point temperature of the metal.

$T_M^{Al} \approx 658$ °C $\qquad T_M^{Mg} \approx 651$ °C.

y_L is the same in both cases. The values above and $T_o = 20$ °C are inserted into equation (85), which gives

$$\frac{t_{\mathrm{Al}}}{t_{\mathrm{Mg}}} = \frac{\left(\dfrac{\rho_{\mathrm{metal}}\left(-H\right)}{T_{\mathrm{M}} - T_{\mathrm{o}}}\right)_{\mathrm{Al}}}{\left(\dfrac{\rho_{\mathrm{metal}}\left(-\Delta H\right)}{T_{\mathrm{M}} - T_{\mathrm{o}}}\right)_{\mathrm{Mg}}} = \frac{2.69 \cdot 10^{3} \cdot 354 \cdot 10^{3}}{658 - 20} \cdot \frac{651 - 20}{1.74 \cdot 10^{3} \cdot 208 \cdot 10^{3}} = 2.6$$

Answer: The Mg alloy should be chosen. It has the shortest solidification time.
