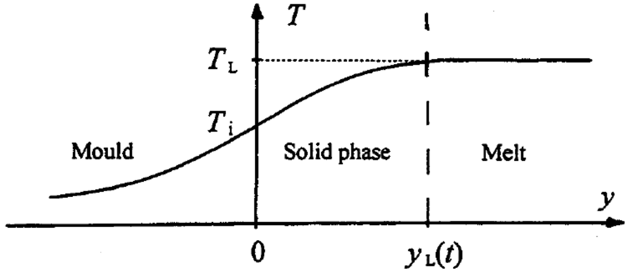
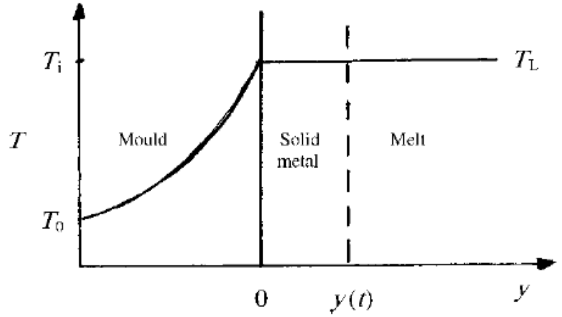
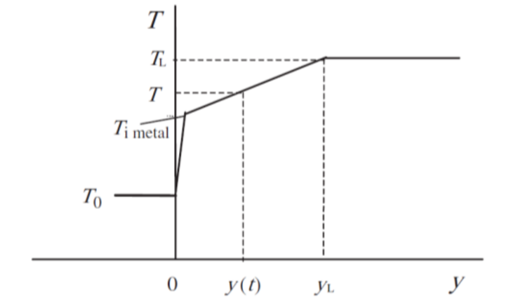
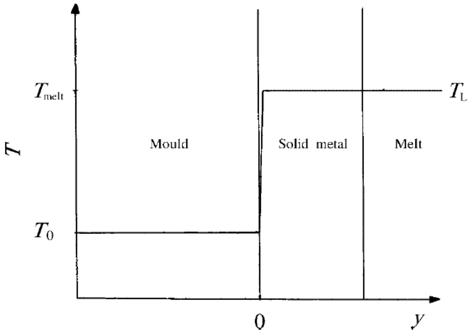
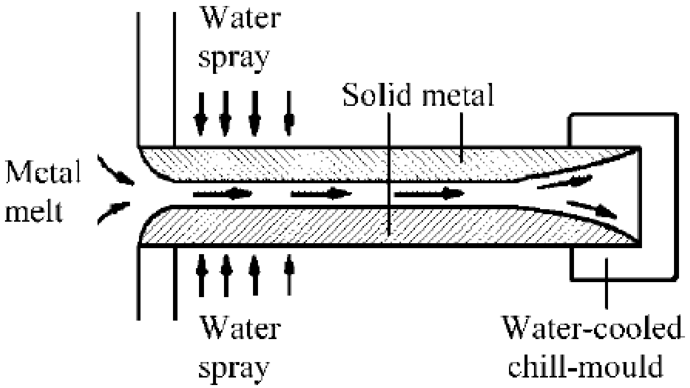
<table>
<tr>
<td rowspan="2">Ideal contact between mould and metal</td>
<td>General case
$$\boxed{y_L(t) = \lambda \cdot \sqrt{4\alpha_{\text{metal}} t}}$$
$$\frac{c_p^{\text{metal}}(T_L - T_0)}{-\Delta H} = \sqrt{\pi}\,\lambda \exp^{(\lambda^2)}\left(\sqrt{\frac{k_{\text{metal}}\rho_{\text{metal}}c_p^{\text{metal}}}{k_{\text{mould}}\rho_{\text{mould}}c_p^{\text{mould}}}} + \operatorname{erf}\lambda\right)$$</td>
<td>
</td>
</tr>
<tr>
<td>Special case: Poor conductivity of mould:
sand casting: assume $T_L = T_i$
$$y_L(t) = \frac{2}{\sqrt{\pi}}\frac{T_i - T_0}{\rho_{\text{metal}}(-\Delta H)}\sqrt{k_{\text{mould}}\rho_{\text{mould}}c_p^{\text{mould}}}\sqrt{t}$$
$$t_{\text{total}} = C\left(\frac{V_{\text{metal}}}{A}\right)^2$$ Chvorinov's rule</td>
<td>
</td>
</tr>
<tr>
<td rowspan="2">Poor contact (air gap is present between mould and melt)</td>
<td>General case
$$t = \frac{\rho(-\Delta H)}{(T_L - T_0)}\left(\frac{y_L}{h}\right)\left(1 + \left(\frac{h}{2k}\right)y_L\right)$$</td>
<td>
</td>
</tr>
<tr>
<td>Special case: Nu<<1 (<0.1)
• Low heat transfer coefficient at mould-solid interface
• Thin solid thickness
• High thermal conductivity of metal
$$t = \frac{\rho(-\Delta H)}{(T_L - T_0)}\left(\frac{y_L}{h}\right)$$</td>
<td>
</td>
</tr>
</table>

Problem 5.2

5.2 The sketch below illustrates the so-called Watts' continuous casting process for steel slabs. A critical part of the process is to keep a sufficiently large part of the central material at the inlet molten during the whole casting process. This can be controlled using an excess temperature of the melt. The excess temperature is required to keep the channel continuously open.



(a) Derive a relationship that shows how the shell thickness varies with excess temperature of the melt under stationary conditions.

Hint A55

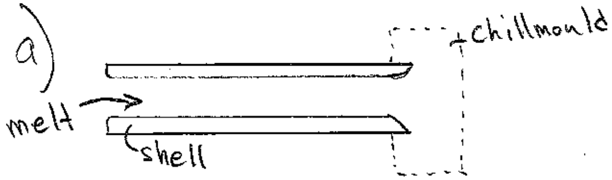
(b) What is the minimum excess temperature of the melt that has to be used if you want to cast slabs of thickness 20 cm in the machine?

Hint A129

Assume that the heat transfer coefficient between the melt and the solidified material is constant. The width of the casting is much larger than its thickness.

The liquidus temperature of the steel is 1450 °C. The heat transfer coefficient between the slab surface and the cooling water is 1.0 kW/m^2 K and between the steel melt and solid phase it is 0.80 kW/m^2 K. The thermal conductivity of steel is 30 W/m K.

5.2) a)



Set up heat transport equations at a cross-section.

From melt to solid phase

$$\frac{dq}{dt} = -h_2(T_{melt} - T_L) \quad ①$$

Flux through solidified shell

$$\frac{dq}{dt} = -k_s\left(\frac{T_L - T_{i,metal}}{y_L}\right) \quad ②$$

From mould interface to surroundings

$$\frac{dq}{dt} = -h_1(T_{i,metal} - T_0) \quad ③$$

First combine ① and ②

$$-h_2(T_{melt} - T_L) = -k_s\frac{(T_L - T_{i,metal})}{y_L}$$

$$y_L = \frac{k_s}{h_2}\frac{(T_L - T_{i,metal})}{(T_{melt} - T_L)} \quad ④$$

$(T_{melt} - T_L) = \Delta T_{excess}$

$T_{i,metal}$ still unknown

Combine ② and ③

$$k_s\frac{(T_L - T_{i,metal})}{y} = h_1(T_{i,metal} - T_0) \Rightarrow T_L - T_{i,metal} = \frac{h_1 y}{k_s}(T_{i,metal} - T_0) \; + T_0 - T_0$$

$$T_L - T_0 = \frac{h_1 y}{k_s}T_{i,metal} - \frac{h_1 y}{k_s}T_0 + T_{i,metal} - T_0 = (T_{i,metal} - T_0)\left(1 + \frac{h_1 y}{k}\right)$$

$$\frac{T_L - T_0}{1 + \frac{h_1 y}{k}} = T_{i,metal} - T_0 \Rightarrow T_{i,metal} = \frac{T_L - T_0}{1 + \frac{h_1 y}{k}} + T_0 \quad ⑤$$

Combine ④ and ⑤

$$y = \frac{k_s}{h_2} \cdot \frac{T_L - \left(\frac{T_L - T_0}{1 + \frac{h_1 y}{k_s}} + T_0\right)}{\Delta T_{excess}} \Rightarrow$$

reduced to

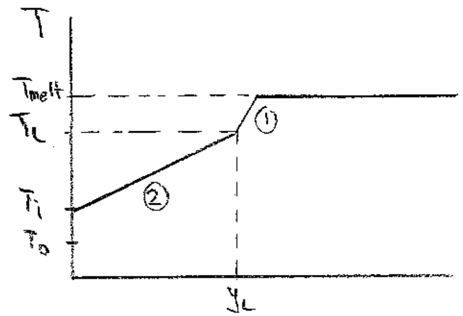
$$\underline{y = \frac{k_s}{h_1}\left(\frac{h_1}{h_2}\frac{T_L - T_0}{\Delta T} - 1\right)} \quad ⑥$$

b) Reorder ⑥

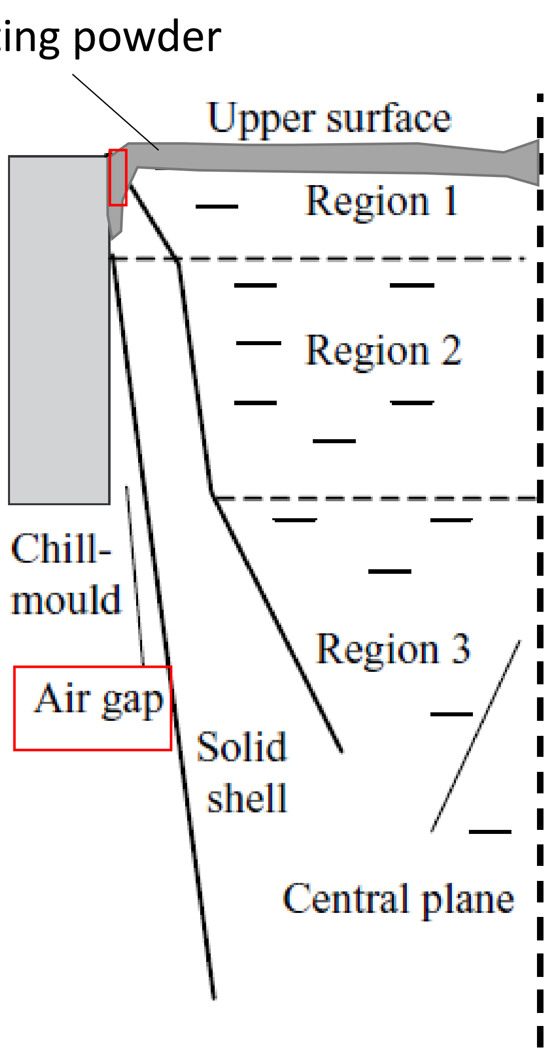
$$\Delta H = \frac{h_1}{h_2}\frac{T_L - T_0}{1 + \frac{h_1 y}{k_s}}$$, with $T_0 = 100°C$ and $y_L = 10cm \Rightarrow \Delta H \approx 400°C$

In this case T_{melt} needs to be at least 1850°C..

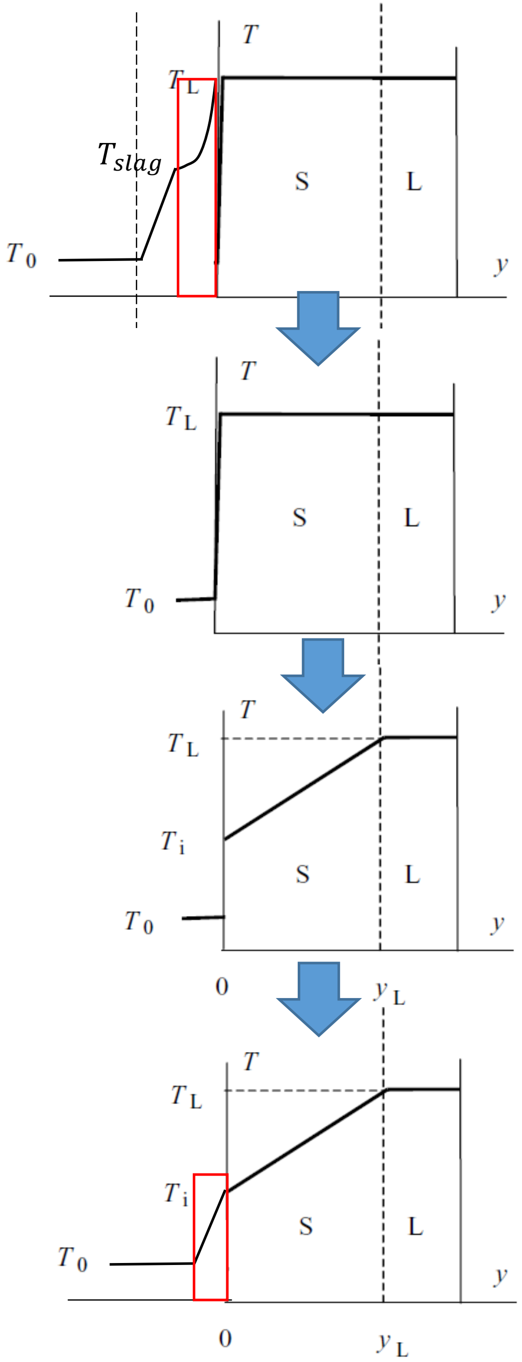
In reality, as the channel width decrease, the velocity of the melt increase. Higher melt velocity increases the heat transfer coefficient, h_2. This counters the need for such high ΔT_{excess}.



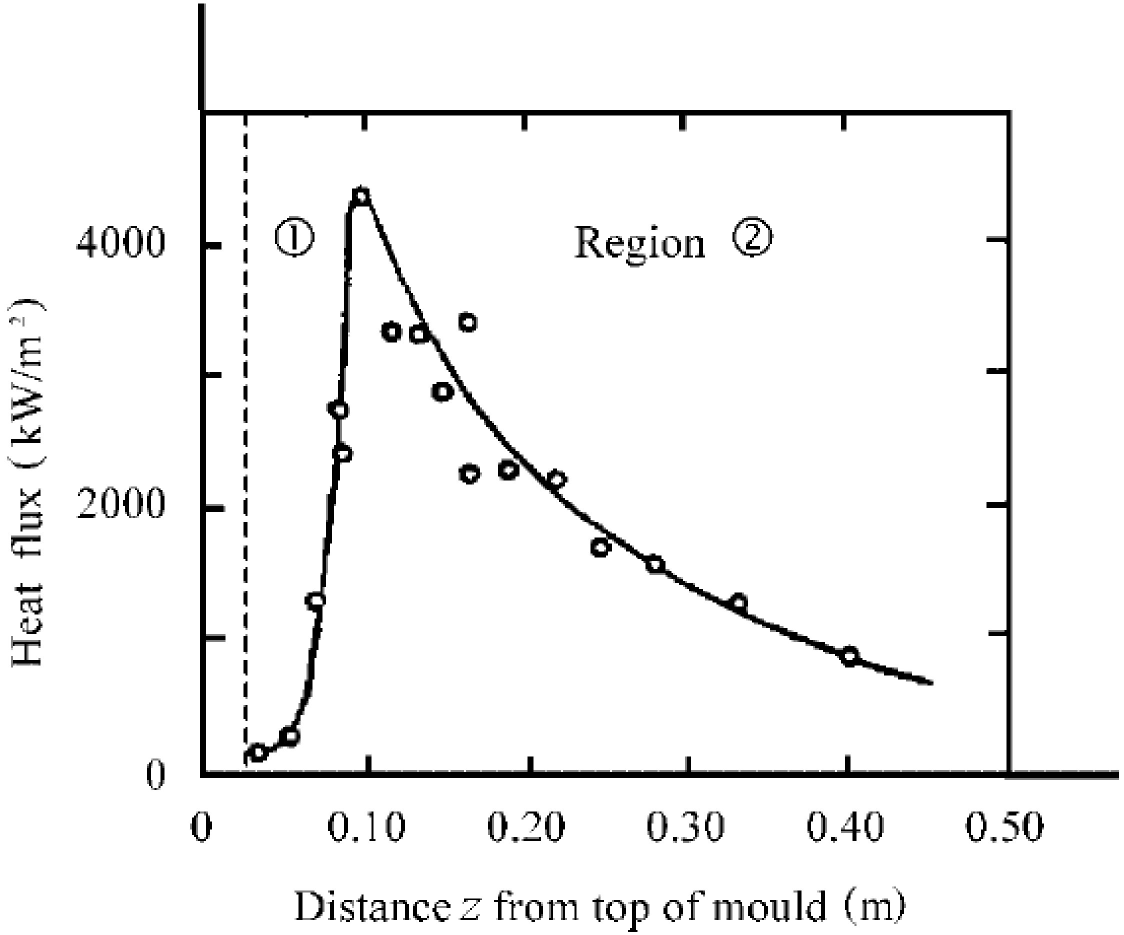
Problem 5.4



Casting powder
Upper surface
Region 1
Region 2
Region 3
Chill-mould
Air gap
Solid shell
Central plane

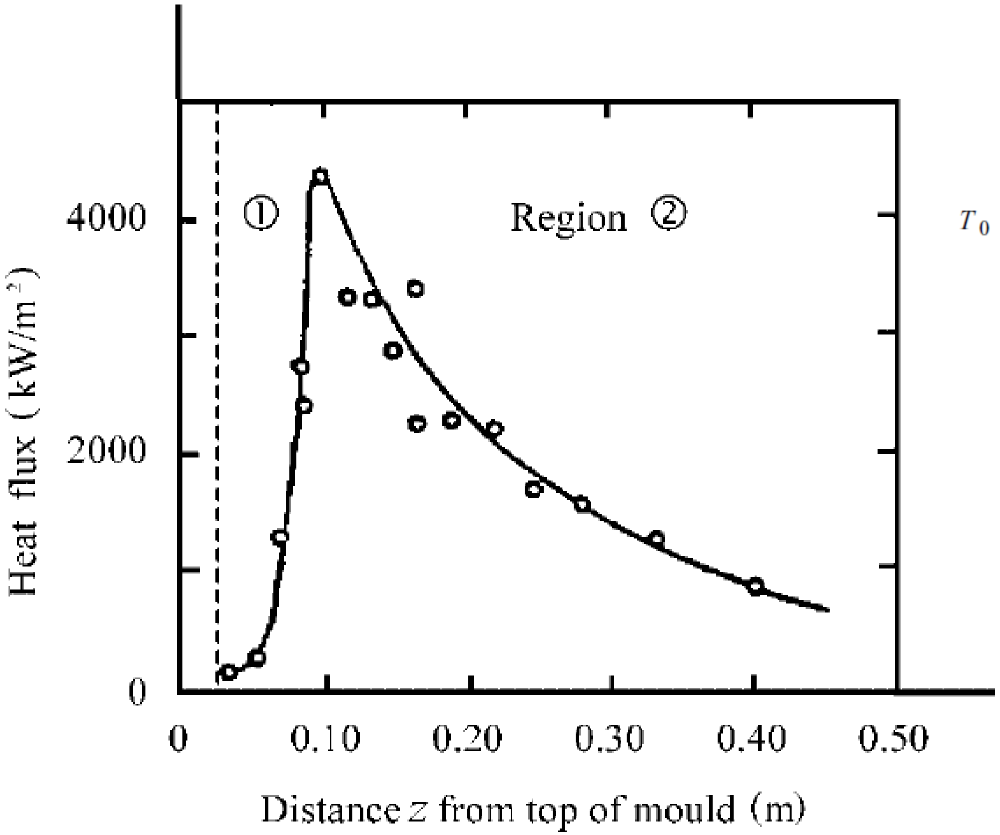


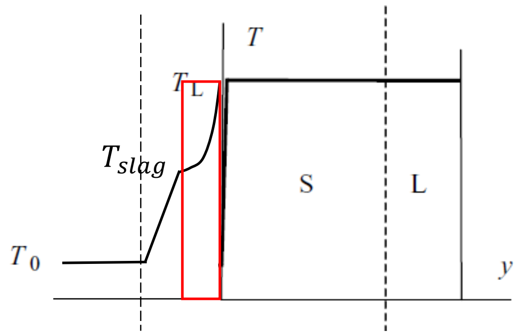
T
T_L
T_{slag}
T_0
S
L
y
T_i
0
y_L

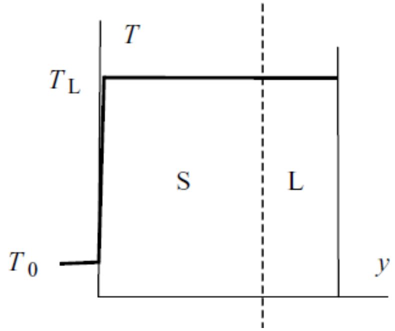


4000
2000
0
① Region ②
Heat flux (kW/m²)
0 0.10 0.20 0.30 0.40 0.50
Distance z from top of mould (m)

Problem 5.4







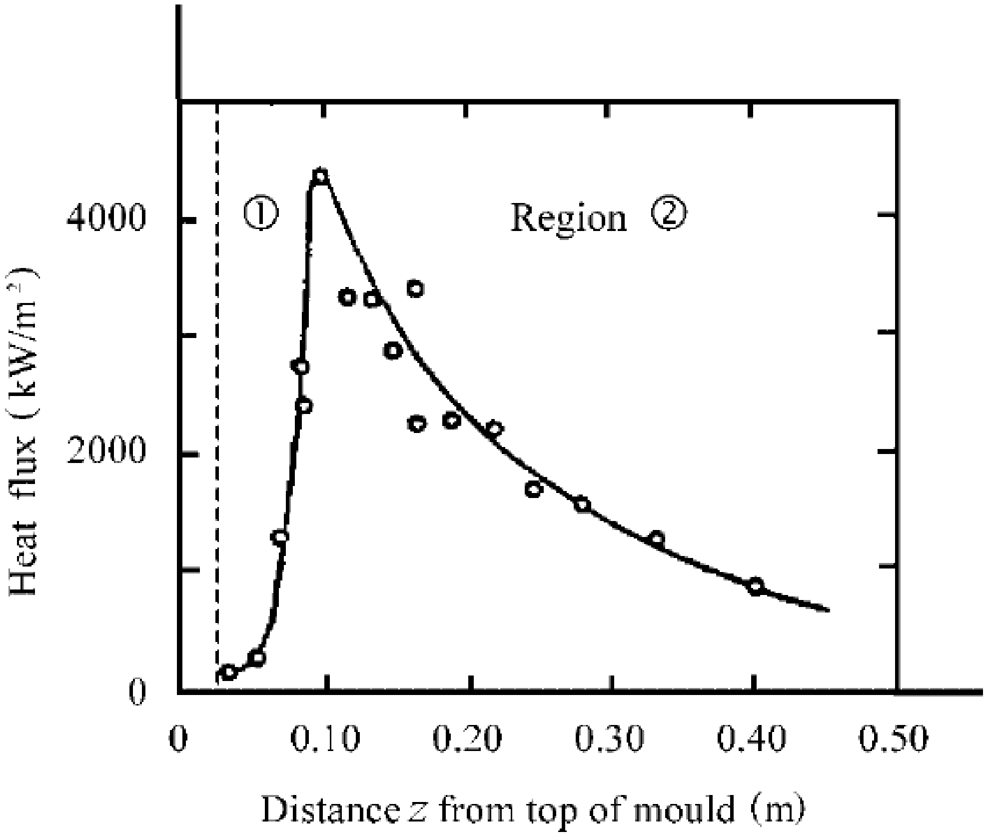
Initially, the casting powder (and its molten state the slag) creates a thin film that lowers the heat transfer. This film decreases with increasing distance from the top of the melt.
This increases the heat transfer which explains the increasing slope of heat flux in region 1.

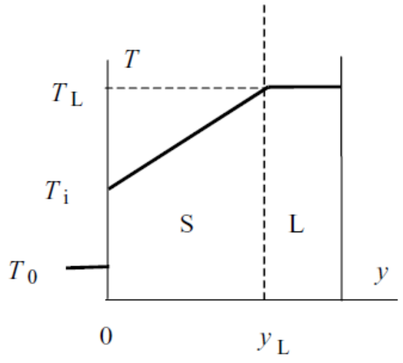
$$\frac{dq}{dt} = -h(T_i - T_0)$$

At the maximum heat flux, the isolating fillm is gone and there is a direct contact of solid shell with the mould. Good contact is realized as the ferrostatic pressure of the melt presses the shell against the mould.

$$t = \frac{\rho(-\Delta H)}{(T_L - T_0)}\left(\frac{y_L}{h}\right)\left(1 + \left(\frac{h}{2k}\right)y_L\right)$$

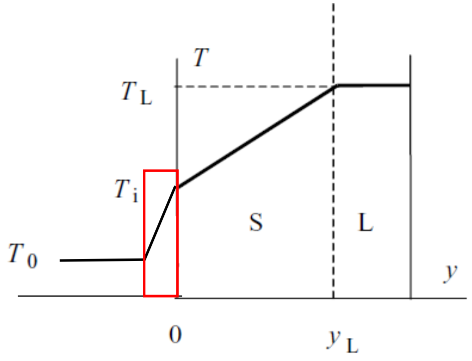
Problem 5.4





As the thickness of the shell increases, the temperature at the mould-shell interface decreases (see result of exercise 4.9), and thus the temperature gradient in the solid is reduced. This then reduces the heat flux at shell-mould interface.

$$\frac{dq}{dt} = -h(T_i - T_0)$$

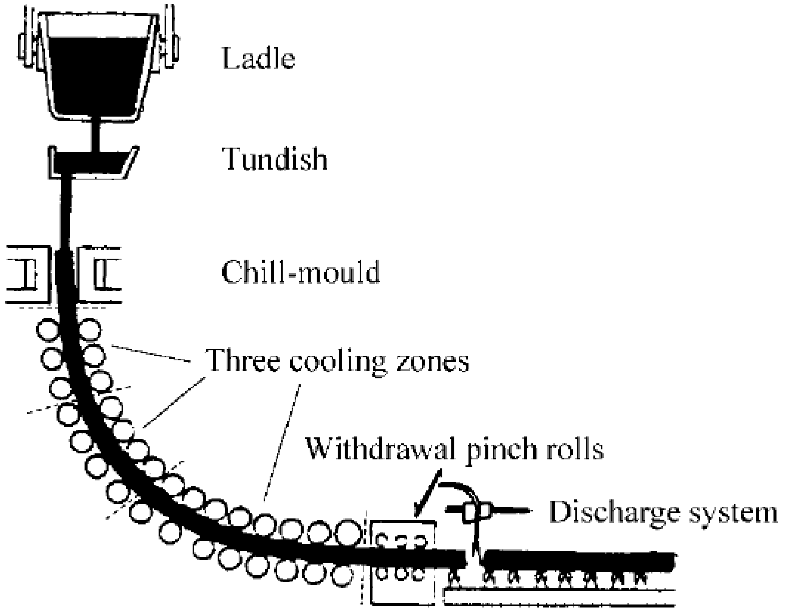


When the shell thickness has grown to a critical level such that it can resist the ferrostatic pressure, an air gap is formed. This significantly lowers the heat transfer at the shell surface and causes further decrease in heat flux.

$$\frac{dq}{dt} = -h(T_i - T_0)$$

Problem 5.5

5.5 The figure below shows a sketch of a continuous casting machine, especially designed for steel. The cross-section area of the casting is 1500 mm × 290 mm. mm. The machine contains three secondary cooling zones.



Cooling zone	Heat transfer numbers ($W/m^2 K$)	Zone length (m)
Chill-mould	1000	1.0
1	440	4.0
2	300	5.0
3	200	10.0

Material constants	
ρ	7.88×10^3 kg/m^3
$-\Delta H$	272 kJ/kg
k	30 W/mK
T_L	1470 °C

Calculate

(a) the total solidification time;

Hint A104

(b) the maximum casting rate

Hint A147

for continuous casting of steel using the machine.

Information about the cooling zones and material constants are given in the tables. The solution can be simplified by calculation of a 'weighted' average value of the heat transfer coefficient for the whole machine.

5.5 a) Total solidification time:

Calculate a weigthed average value of h.

$\frac{dQ}{dt} = h \cdot A \cdot (T - T_0)$, area proportional to the length of each zone

$$h_{av} = \frac{h_1 L_1 + h_2 L_2 + h_3 L_3 + h_4 L_4}{L_1 + L_2 + L_3 + L_4} = 313\ W/m^2K$$

Poor contact and $Nu = \frac{hs}{k} = \frac{313 \cdot 0.290}{30 \cdot 2} = \underline{1.52}$
Heat conduction through solidified melt can't be neglected.

$$t = \frac{\rho(-\Delta H)}{T_L - T_0} \cdot \left(\frac{y_L}{h} + \frac{y_L^2}{2k}\right)$$, all except T_0 is known

Since water spray is used, we assume $T_0 = 100°C$

$$t = \frac{7.88 \cdot 10^3 \cdot 272 \cdot 10^3}{1470 - 100} \cdot \left(\frac{0.145}{313} + \frac{0.145^2}{2 \cdot 30}\right) = 1273\,s \sim \underline{21\,min}$$

b) What is the maximum casting rate?

The casting must be solidified before the end of the last cooling zone.

$$V_{cast} = \frac{L_{total}}{t} = \frac{20}{21} = \underline{0.95\ m/min}$$

Problem 5.6

5.6 During casting in a certain machine for continuous casting the following data are valid:

Dimension of casting $a \times a$:	100 mm × 100 mm
Casting rate:	3.0 m/min
Temperature of cooling water:	40 °C.

The data for the cooling zones are given in the table. Calculate the heat transfer coefficient between the metal surface and the cooling water for each zone.

Hint A15

Cooling zone	Length (mm)	Water flow (litre/min)
Spray zone	200	80
Zone 1	1280	175
Zone 2	1850	150
Zone 3	1900	175

5.6) Calculate heat transfer coefficient for all cooling zones.

Use empirical eq.(5.30)

$$h_w = \frac{1.57 \cdot w^{0.55}(1-0.0075 \cdot T_w) \cdot 10^3}{\alpha}$$

α: machine dependent, around 4
w: water flux in litre/m²·s
T_w: cooling water temperature

For spray zone:

$$W_{sprayzone} = \frac{flow}{A} = \left\{A = \text{Area of all sides}\right\} = \frac{80}{0.2 \cdot 0.1 \cdot 4} \cdot \frac{1}{60} = 16.6 \ \ell/m^2 \cdot s$$

$$h_w = \frac{1.57 \cdot 16.6^{0.55}(1-0.0075 \cdot 40) \cdot 10^3}{4} = \underline{1288 \ W/m^2 \cdot K}$$

Problem 5.6

	Q_{water} (L/min)	Q_{water} (L/sec)	L_{zone} (m)	w	$h_w\ (\frac{W}{m^2K})$
Spraying zone	80	1.333	0.2	16.625	1288
Zone 1	175	2.9167	1.28	5.696	715
Zone 2	150	2.5	1.85	3.378	536.67
Zone 3	175	2.9167	1.9	3.83	575

Problem 5.9

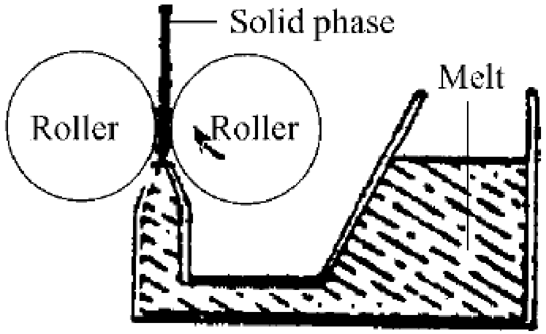
5.9 Aluminium foils are often cast according to the so-called Hunter method, where the melt is forced in between two rotating rollers (upper figure) and brought to solidify between the rollers during a simultaneous movement upwards.

The lower figure (turned 90° relative to the upper figure) illustrates the solidification process of the melt between the rollers. The figure shows that the solidification fronts will meet where the roller slit is thinnest. The consequence of this is that the width of the roller slit will correspond to the maximum thickness of the casting.

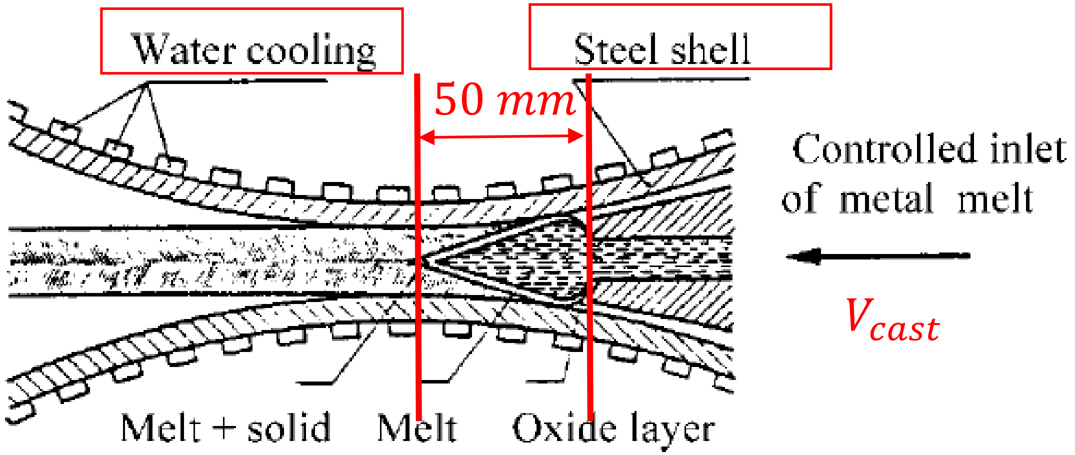
The rollers are very large, i.e. no attention need be paid to the curvature of the rollers in the calculations. The heat transfer number between the casting and the roller is 3.0 kW/m^2 K. The distance between the exit of the nozzle and the point where the solidification fronts meet is 50 mm.

Calculate the maximum casting rate (m/min) as a function of the thickness of the casting and illustrate the function graphically.

Hint A150



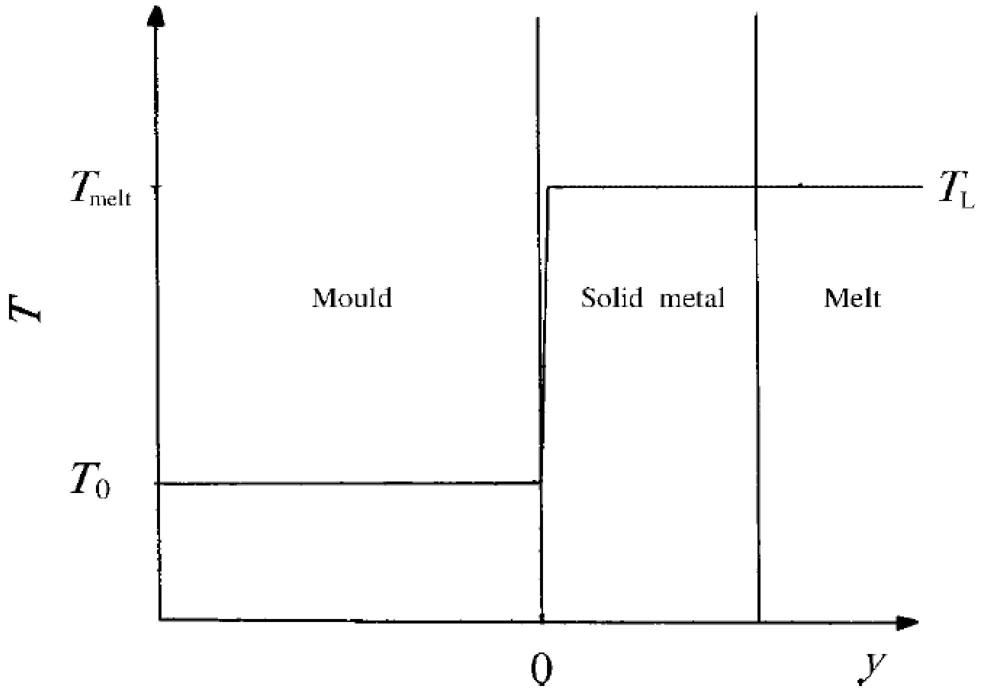
Hunter engineering



Problem 5.9

$$t = \frac{\rho\,(-\Delta H)}{(T_L - T_0)}\left(\frac{y_L}{h}\right)\left(1 + \left(\cancel{\frac{h}{2k}}\right) y_L\right)$$

Material constants for aluminium	
T_L	660 °C
k	220 W/m K at T_L
$-\Delta H$	398 kJ/kg
ρ_s	2.7×10^3 kg/m^3



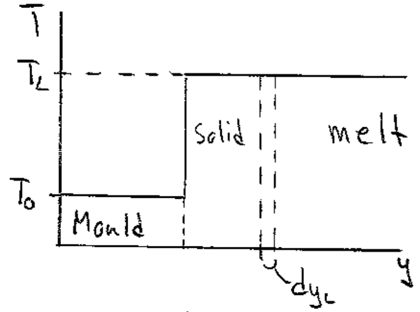
$$V_{cast} = \frac{50\ mm}{t}$$

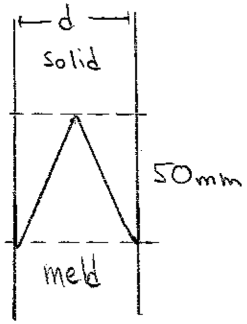
5.9) Calculate maximum casting rate as function of foil thickness

Check Nu and set up temp. distribution.

$$Nu = \frac{hs}{k} = \frac{3\cdot 10^3}{220}\cdot s \left\{ \begin{matrix} s < 1\,\text{mm} \\ \text{since foil} \end{matrix} \right\}, \quad Nu < 0.014 \ll 1$$

No conduction in solidified metal





Set up heat balance

$$\begin{pmatrix} \text{Heat from metal} \\ \text{to rollers} \end{pmatrix} = \begin{pmatrix} \text{solidification} \\ \text{heat} \end{pmatrix}$$

$$h(T_L - T_0) = \rho(-\Delta H)\frac{dy_L}{dt}$$

$$h(T_L - T_0)\int_0^t dt = \rho(-\Delta H)\int_0^{y_L} dy_L$$

$$t = \frac{\rho(-\Delta H)}{h(T_L - T_0)}\cdot y_L, \quad y_L = \frac{d}{2}$$

$$V_{cast} = \frac{l}{t}, \quad l = \text{metallurgical length} = 50\,\text{mm}$$

$$V_{cast} = \frac{h(T_L - T_0)}{\rho(-\Delta H)}\cdot\frac{2}{d}\cdot l$$

$$V_{cast} = \frac{3\cdot 10^3(660-100)}{2.7\cdot 10^3\cdot 398\cdot 10^3}\cdot\frac{0.05\cdot 2}{d} = 1.56\cdot 10^{-4}\cdot\frac{1}{d}\,[\text{m/s}] = \underline{0.009\cdot\frac{1}{d}\,[\text{m/min}]}$$

For $d = 5\cdot 10^{-4}$, $V_{cast} = 18$ /min