Problem 10.1

10.1 (a) A steel cube with a side of 20 cm is to be cast. A cylindrical feeder with a diameter of 20 cm is located on top of the cubic mould. Calculate the minimum height of the feeder if the feeder and the mould are made of sand.

Hint B36

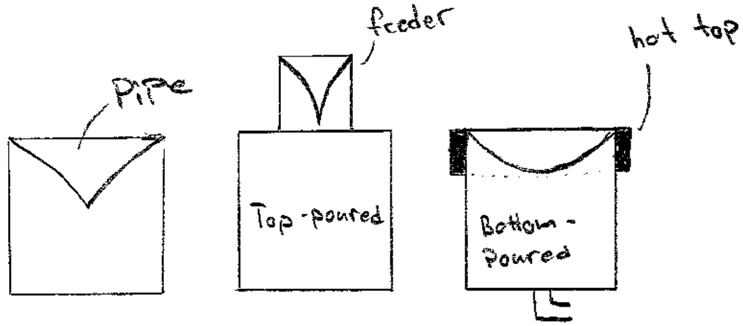
(b) Alternatively, the cube can be cast in the sand mould with a cylindrical feeder, which is made of a more insulating material and has a diameter of 10 cm. In this case the solidification time is prolonged by a factor of four compared with that in (a). Calculate the minimum height of the new feeder. Which alternative is the best one?

Hint B101

Use Table 10.1 on page 316 to find a reasonable value of the solidification shrinkage.

10.1) Designing a feeder or hot top.

The purpose is to avoid pipe formation due to shrinkage.



A succesful feeder or hot top must fulfill two conditions:

1. The solidification time must be longer than the solidification time in the casting.
   The remaining melt in the feeder or hot top can then compensate for the solidification shrinkage in the casting.

   $$(t_{casting} = t_{feeder})$$

2. The volume of the feeder or hot top must be big enough to accomodate the shrinkage in both the casting and itself.

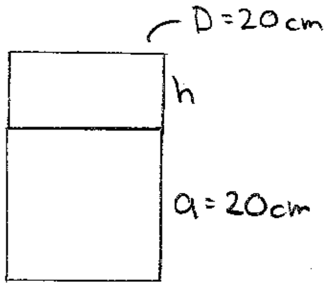
   Meaning:

   $$\begin{pmatrix}\text{Volume of}\\ \text{feeder}\end{pmatrix} = \begin{pmatrix}\text{solidification shrinkage}\\ \text{in feeder and casting}\end{pmatrix} + \begin{pmatrix}\text{Solidified melt}\\ \text{in the feeder}\end{pmatrix}$$

   $$V_f = \beta(V_c + V_f) + V_{sm}$$

   β : solidification shrinkage value

10.1 a) Steel cube in a sand mould. Feeder is of the same material as the mould.



The solidification time in a sand mould can be expressed by the help of Chvorinovs rule.

$$t = C\left(\frac{V}{A}\right)^2$$

First condition gives that $t_f = t_c$

so,

$$C_c\left(\frac{V_c}{A_c}\right)^2 = C_f\left(\frac{V_{sm}}{A_f}\right)^2$$, V_{sm} : volume of solidified melt

From the second condition we get:

$$V_{sm} = V_f - \beta(V_c + V_f)$$

If combined we get:

$$C_c\left(\frac{V_c}{A_c}\right)^2 = C_f\left(\frac{V_f - \beta(V_c+V_f)}{A_f}\right)^2$$

or

$$\frac{V_f}{V_c}(1-\beta) = \left(\frac{C_c}{C_f}\right)^{1/2}\frac{A_f}{A_c} + \beta$$

Insert correct dimensions

$$\frac{\frac{\pi D^2}{4}\cdot h_f}{a^3}(1-\beta) = \left(\frac{C_c}{C_f}\right)^{1/2}\frac{\pi D}{6a^2 - \frac{\pi D^2}{4}}\cdot h_f + \beta$$

In this case, since mould and feeder have the same material, $C_c/C_f = 1$.

From table 10.1 on page 316 we get $\beta = 0.04$.

$$\frac{\pi 0.2^2}{4\cdot 0.2^3}\cdot 0.96 = \frac{\pi\cdot 0.2}{6\cdot 0.2^2 - \frac{\pi\cdot 0.2^2}{4}} + \frac{0.04}{h_f}$$

$h_f = 0.053 \sim \underline{5\text{cm}}$

10.1 b) Alternative feeder has 4 times longer solidification time compared to the feeder in a). The insulated feeder has a diameter of 10 cm.

We can use the same equation we derived in a).

$$\frac{V_f}{V_c}(1-\beta) = \left(\frac{c_c}{c_f}\right)^{1/2} \frac{A_f}{A_c} + \beta$$

insert values and $c_c/c_f = 1/4$

$$\frac{\pi D^2}{4a^3} \cdot h_f(1-\beta) = \left(\frac{c_c}{c_f}\right)^{1/2} \frac{\pi D}{6a^2 - \frac{\pi D^2}{4}} \cdot h_f + \beta \qquad \{c_f = 4 \cdot c_c\}$$

$$\frac{\pi \cdot 0.1^2}{4 \cdot 0.2^3} \cdot 0.96 = \frac{1}{2} \cdot \frac{\pi \cdot 0.1}{6 \cdot 0.2^2 - \frac{\pi \cdot 0.1^2}{4}} + \frac{0.04}{h_f}$$

$$h_f = 0.150\,m = \underline{15\,cm}$$

Which feeder should be used?

$V_f = 0.0017\,m^3$ and $V_{f,insulated} = 0.0012\,m^3$

$V_{sm} = 1.3 \cdot 10^{-3}\,m^3$ and $V_{sm,insulated} = 8.3 \cdot 10^{-4}\,m^3$

The wasted material in the insulated feeder is very little compared to the other.

Choose the insulated feeder!

Problem 10.2

10.2 A hollow cylinder with the dimensions external diameter 40 cm, internal diameter 20 cm and height 30 cm is to be cast in a copper alloy. The cylinder can be cast in either brass or Al-bronze.

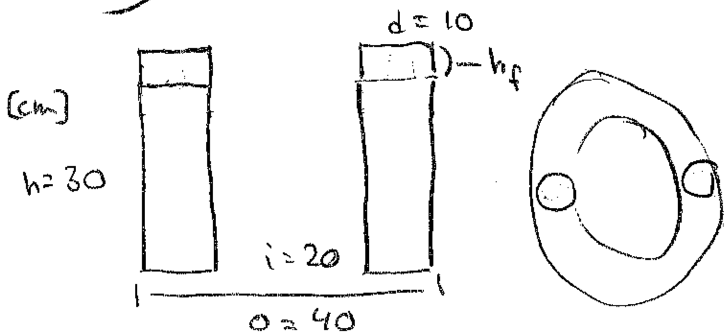
A demand on the cylinder is that it has to be completely compact. For this reason, two cylindrical feeders, each with a diameter of 10 cm, are chosen. The material, size and design of the feeders determine the choice of alloy. The size of the feeder will be chosen to be as small as possible.

The cylinder mould is made of sand and the feeders are made of a highly insulating ceramic material. Material constants of the mould and feeder materials are given in the table here. Tables 10.1 (page 316) and 10.2 (page 326) can be used to find the required material constants of the alloys.

Calculate the height of the feeders in the two alternatives and suggest a choice of material.

Material constants	
Cylinder sand:	
k_c	14.5×10^{-4} J/m s K
ρ_c	1.5×10^{3} kg/m^3
c_p^c	0.27 kJ/kg K
Feeder material	
k_f	4.1×10^{-4} J/m s K
ρ_f	0.90×10^{3} kg/m^3
c_p^f	0.20 kJ/kg K

10.2)



Two feeders to ensure no pipe and to keep the size of the feeders small

Time condition : $t_{feeder} = t_{cast}$ (usually)

Since [illegible] casting an alloy

Time condition is $t_{cast} = t_{bsof} = t_f \cdot \left(1 - \frac{CFR}{100}\right)$

t_{bsof}: time of the beginning solidification in the centre of the feeder

CFR : Centreline feeding resistance (Table 10.2 p.325)

$CFR_{Brass} = 26$ $\quad CFR_{Al\text{-}bronze} = 95$

$$\frac{V_f}{V_c}(1-\beta) = \left(\frac{C_c}{C_f\left(1-\frac{CFR}{100}\right)}\right)^{1/2} \frac{A_f}{A_c} + \beta$$

From Table 10.1 p. 316 ; $\beta_{brass} = 0.05$, $\beta_{Al\text{-}bronze} = 0.05$

Calculate C_c and C_f !

$\left(\frac{C_c}{C_f}\right)_{use} = \left(\frac{k_c \rho_c c_p^c}{k_c \rho_c c_p^c}\right)$, independent of the material constants of the cast alloy

$\left(\frac{C_c}{C_f}\right) = 0.126$

Calculate the heigth, h_f !

Set up expression for V_f, V_c, A_f, A_c

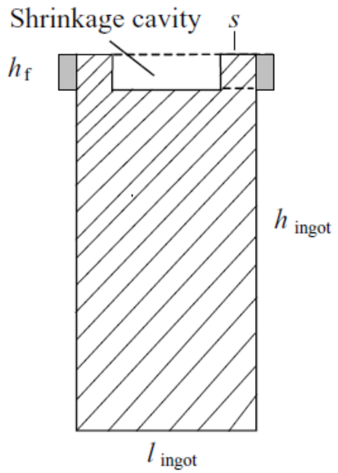
$V_f = \frac{\pi d^2}{4} \cdot h_f \cdot 2$, $V_c = \left(\frac{\pi o^2}{4} - \frac{\pi i^2}{4}\right) 0.3$, $A_f = \pi d \cdot h_f$

$$A_c = \underbrace{(\pi o + \pi i) \cdot 0.3}_{\text{walls}} + \underbrace{\left(\pi\left(\frac{o}{2}\right)^2 - \pi\left(\frac{i}{2}\right)^2\right) \cdot 2}_{\text{top and bottom}} - \underbrace{\pi\left(\frac{d}{2}\right)^2 \cdot 2}_{\text{feeder entrance}}$$

Problem 10.4

10.4 An ingot with dimensions 0.20×1.00 m and a height of 1.30 m is to be cast in steel in a thick iron mould. The ingot is equipped with a hot top made of sand with the same cross-sectional area as the ingot. Immediately after casting, the upper surface of the ingot is covered with a layer of 30 mm of silica sand as heat insulation.

However, it is doubtful whether the height of the hot top has been chosen properly for satisfactory use. For this reason, it is important to calculate the minimum value of the height of the hot top.



(a) As a first rough estimation, the following simple but rather unrealistic method can be used.

The total shrinkage volume is assumed to be equal to the hollow volume of the hot top. Perform an approximate calculation of the height h_f of the hot top if the thickness s of the solidified shell in the hot top is guessed to be 15 mm at the time of total solidification of the ingot.

Calculate the height of the hot top with the aid of this method.

Material constants:

Steel:	*Hot top made of sand:*
$\rho = 7.5 \times 10^3$ kg/m^3	$\rho = 1.6 \times 10^3$ kg/m^3
k = 30 W/m K	$k = 0.63$ W/m K
$-\Delta H = 272 \times 10^3$ J/kg	$c_p = 1.05 \times 10^3$ J/kg K
$T_L = 1550\,°C$	

Problem 10.4

10.4 An ingot with dimensions 0.20×1.00 m and a height of 1.30 m is to be cast in steel in a thick iron mould. The ingot is equipped with a hot top made of sand with the same cross-sectional area as the ingot. Immediately after casting, the upper surface of the ingot is covered with a layer of 30 mm of silica sand as heat insulation.

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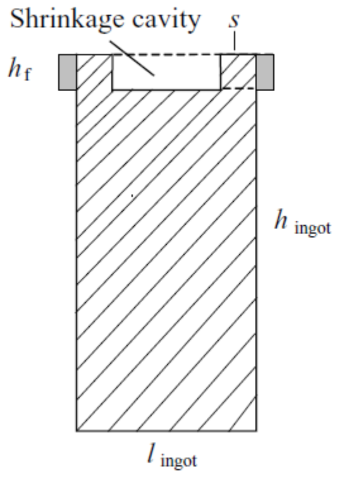


(b) The method in (a) gives a too low value of the hot top as it must be large enough to keep some melt even after the solidification of the ingot.

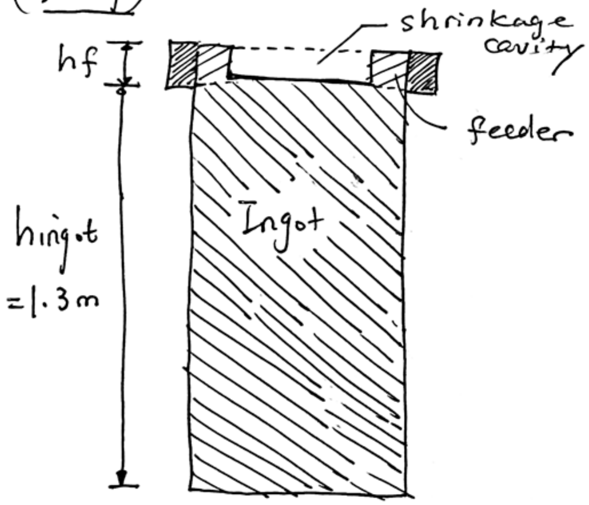
Perform a more accurate calculation of the height of the hot top. The information that the thickness of the shell is 15 mm is no longer valid. The thickness of the solidified shell of the hot top at the time of total solidification of the ingot has to be calculated.

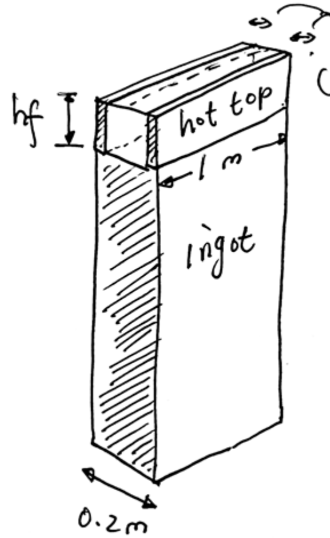
Hint B222

Material constants:	
Steel:	*Hot top made of sand:*
$\rho = 7.5 \times 10^3\ \mathrm{kg/m^3}$	$\rho = 1.6 \times 10^3\ \mathrm{kg/m^3}$
$k = 30\ \mathrm{W/m\,K}$	$k = 0.63\ \mathrm{W/m\,K}$
$-\Delta H = 272 \times 10^3\ \mathrm{J/kg}$	$c_p = 1.05 \times 10^3\ \mathrm{J/kg\,K}$
$T_L = 1550\ ^\circ\mathrm{C}$	

(c) Do you find the method in (a) useful?

(10.4)





$S = 0.015\,m$
$= 15\,mm.$

(a) Consider bilateral solidification as we have done before!

$$\begin{pmatrix}\text{Cavity}\\\text{volume}\end{pmatrix} = \begin{pmatrix}\text{Total shrinkage}\\\text{in hot top and ingot}\end{pmatrix}$$

$$V_{cav} = V_f - V_{sm} = \beta\,(V_c + V_f)$$

Take solidification shrinkage, $\beta = 4\%$.

Thus;

$$h_f(0.2 - 2\times S)\times 1 = (0.04)(1.3\times 0.2\times 1 + h_f\times 2S)$$

$$\Rightarrow h_f \doteq 6.2\,cm.$$

(b) Consider solidification of ingot and hot top separately.

① Use an empirical relation for solidification of ingot : $y^* = 2.5\sqrt{t}$
convert it into SI units, and do so re-arrangement to make it look like Chvorinov's rule. $\begin{pmatrix} y \text{ in cm} \\ t \text{ in minutes}\end{pmatrix}$.

$$t_{ingot} = (9.6\times 10^4).\,y_{ingot}^{\,2} \quad (t \text{ in sec, } y \text{ in } m)$$
$$= C_i \cdot y_{ingot}^{\,2}$$

(10.4) (cont'd)

(b) ② Consider normal Chvorinov's rule: $t_{top} = C_{top}\left(\frac{V_{top}}{A_{top}}\right)^2$.

$$C_{top} = \frac{\pi}{4}\,\frac{\rho_{steel}^2\cdot(-\Delta H_{steel})^2}{(T_i - T_o)^2}\,\frac{1}{\cdot\,\rho_{sand}\cdot C_{p,sand}\cdot K_{sand}} = C_{top}\cdot(y_{top})^2$$

For sand mould casting $T_i = T_L = 1550°C$. Assume $T_o = 20°C$.

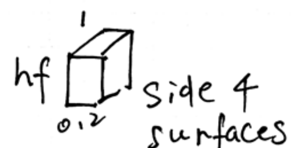
Then $$C_{top} = \frac{\pi}{4}\cdot\frac{(7500)^2\cdot(272.10^3)^2}{(1550 - 20)^2}\cdot\frac{1}{0.63\cdot(1.05\cdot1000)\cdot1600}$$

$$\doteq 1.32\times10^6\ (s/m^2)$$

③ Use the volume criteria to design the feeder size.
(essentially hot top is insulated feeder)

$C_{top} = C_f$, $C_c = C_i$

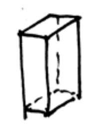
Then $$\frac{V_{top}}{V_i}(1-\beta) = \left(\frac{C_c}{C_f}\right)^{1/2}\left(\frac{A_f}{A_c}\right) + \beta.$$



$$\Rightarrow \frac{h_f\times\cancel{A_i}}{1.3\times\cancel{A_i}}(0.96) = \left(\frac{9.6\times10^4}{1.32\times10^6}\right)^{\frac{1}{2}}\times\left(\frac{2\times(h_f\times0.2 + h_f\times1)}{2\times(0.2\times1.3+1\times1.3)+(0.2\times1)}\right) + (0.04)$$

(ingot and hot top has same cross-section)

side 4 surfaces + 1 bottom.



(10.4) (Cont'd)

(b). Thus, $h_f \doteq 7.3$ cm. which is larger than that estimated in (a)

(c) Probably not very acurate but still quite useful when one doesn't have information about the growth rate relation between the ingot and the hot top.

Calculation of (a) ignores the fact that the hot top and ingot could have different solidification rate while (b) takes care of it. Thus, it could be seen that satisfying the volume condition: $V_f = V_{sm} + \beta (V_f + V_c)$ alone is not sufficient for properly dimensioning the feeder. Instead.

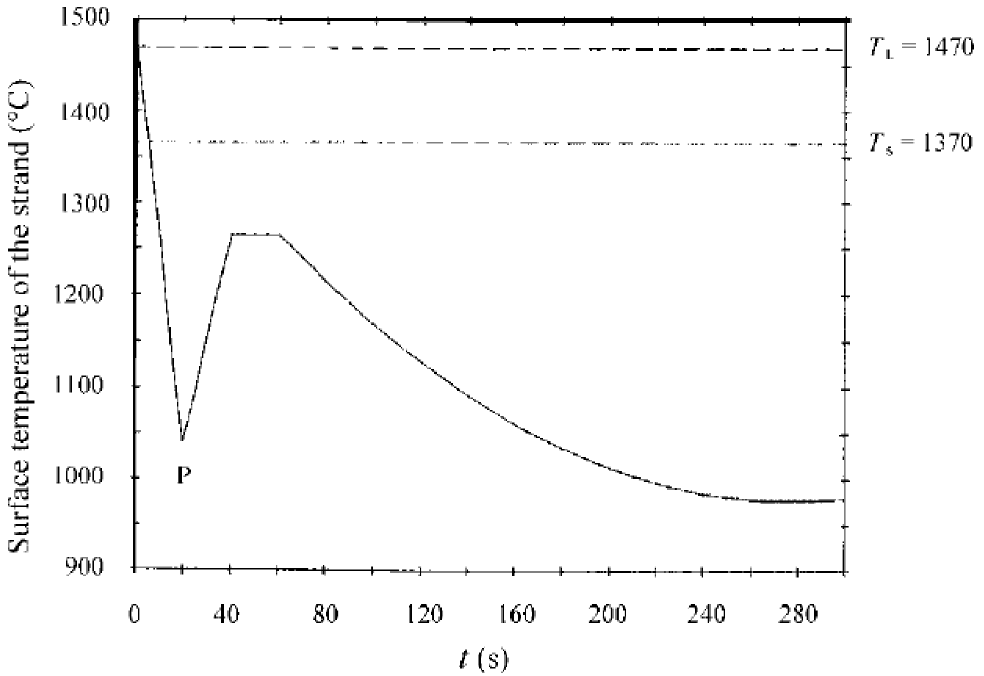
$$\frac{V_f}{V_c}(1-\beta) = \left(\frac{C_c}{C_f}\right)^{1/2}\left(\frac{A_f}{A_c}\right) + \beta$$ is more accurate.

However, C_c and C_f depends on many factors and are not always available.

Problem 10.10

10.10 It has been proposed that the origin of half-way cracks during continuous casting is reheating of the surface when an air gap is formed during the casting process (Figure 10.84 on page 360). In order to confirm or reject this statement, we will analyse the thermal stresses at a continuous casting operation illustrated in the figure shown. It shows the temperature at a point of the strand surface as a function of time.

At the time for reheating of the surface the shell thickness of the strand is 3.0 cm. In order to simplify your calculations, you may assume that the solidification rate is zero during the heating period, that the temperature distribution in the solidified shell is linear and that the shell is free from stresses at point P. You may also assume that Hooke's law is valid. Material constants of the steel alloy are given in the table.



Calculate:

(a) the temperature increase $\Delta T\ (y)$

Hint B15

(b) the strain $\varepsilon\ (y)$ and the stress $\sigma\ (y)$

Hint B56

across the solid shell after the heating period of the surface as functions of the distance y from the surface. Plot them as functions of y in two diagrams.

(c) Plot the stress σ as a function of the temperature and discuss the risk of crack formation.

Problem 10.10

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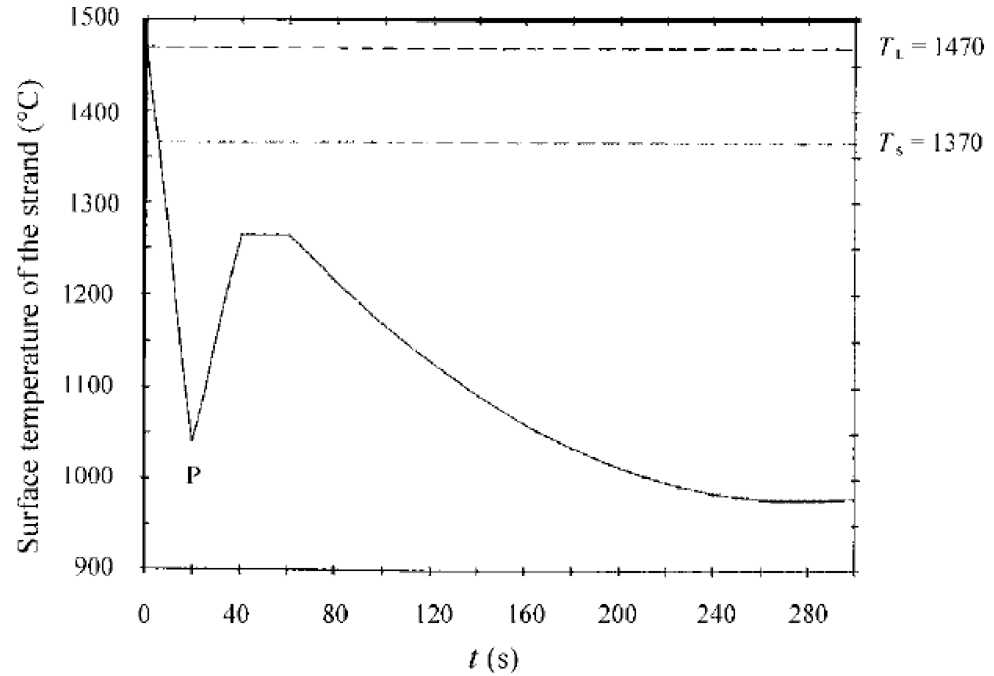
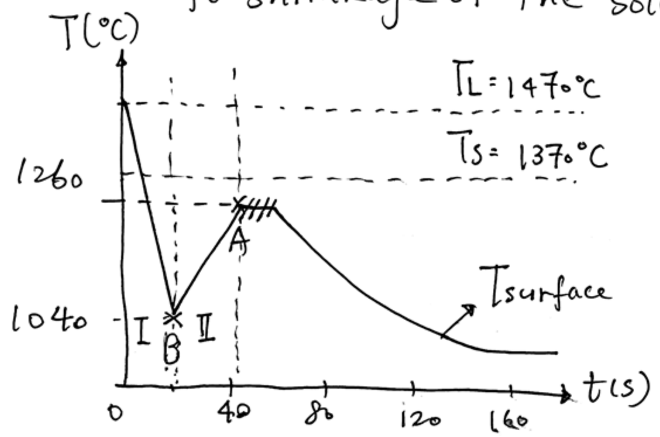


Material constants of the shell	
T_L	1470 °C
T_s	1370 °C
α	5.0×10^{-5} K^{-1}
E	100×10^9 N/m^2
Transition temperature between the ductile and brittle zones:	
T_{tr}	1330 °C

(10.10) The surface of the strand is reheated since an air gap is formed in between the solid shell and the mould. The air gap forms due to shrinkage of the solid shell formed in the copper mould.



(1) During stage I, the melt touches the mould wall and starts to solidify as thin solid shell. The contact between the shell and the mould is good. and the surface of the strand is rapidly cooled to a temperature ~ 1040°C.

(2) During stage II, an air gap starts to form and the heat at the solidification front cannot be transported away quickly such that the surface of the strand heats up to a temperature ~ 1260°C.

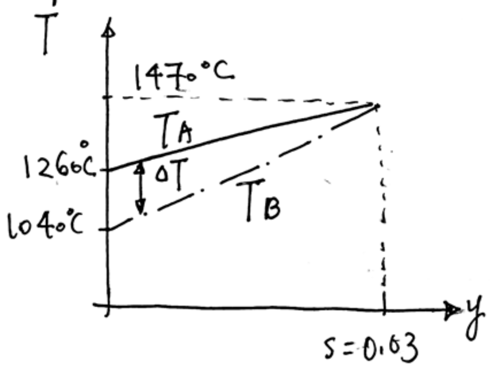
(3) The temperature at the end of stage II is just slightly below the solidus, where the material could be potentially brittle and hot cracks could appear.

Thus, we would focus on the reheating period in stage II.

(10.10) (cont'd)

(4) We can draw the temperature distribution in the solid shell (3 cm thick and according to text, there is no further solidification during reheating period) at end of stage I and II (instant A and B respectively).

(a)



$$T_B(y) = 7000 \cdot y + 1260 \quad (°C)$$

$$T_A(y) = 14333.33\, y + 1040 \quad (°C)$$

Thus, the temperature difference between instant B and A at the surface of the solid shell is: $\Delta T(y) = T_B(y) - T_A(y)$

$$= 220\left(1 - \frac{y}{0.03}\right). \ (°C). \ \#$$

(b) The thermal strain depends on temperature difference. And according to the text the material could be assumed free of stress at instant A, hence, we consider strain purely due to thermal expansion due to reheating in stage II.

We use the relation: (Derivation given at the end).

$$\varepsilon^T(y) = \alpha\left(T_B(y=0) - T_A(y=0)\right)\left(\frac{1}{2} - \frac{y}{0.03}\right).$$

$$= 220\alpha\left(\frac{1}{2} - \frac{y}{0.03}\right).$$

$$= 0.611\left(\frac{1}{2} - \frac{y}{0.03}\right) \quad \#$$

(10.10)(cont'd)

(b) We know from Hooke's law : $\sigma^T(y) = -E\varepsilon^T$.
Why minus sign? Because the material of the solid shell is a part of the whole strand. i.e. net force of different parts of strand must be zero. The surface part which is heated more severely has the tendency to expand, but the rest of the material does want it to, Thus, compressive stress comes as a result of expansion (positive thermal strain). Vice versa for center part of the strand. which is heated less severely, tends to contract a little, and is subjected to tensile stress.

since overall the solid shell is expanding due to positive $\Delta T(y)$. There are regions which expands "more" or "less" than the "average". The regions that expands less than the average would be subjected to negative thermal strain.

Thus:

$$\sigma^T(y) = (-100)\times 10^9 \times (0.011)\left(\frac{1}{2} - \frac{y}{0.03}\right)$$
$$= -1100\left(\frac{1}{2} - \frac{y}{0.03}\right). \text{(MPa)}$$

$GPa = 10^9\ N/m^2$

$MPa = 10^6\ N/m^2$

$Pa = 1\ N/m^2$

(10.10) (cont'd)

(c) Plot the stress σ as a function of y:

The criteria for hot crack formation is that:

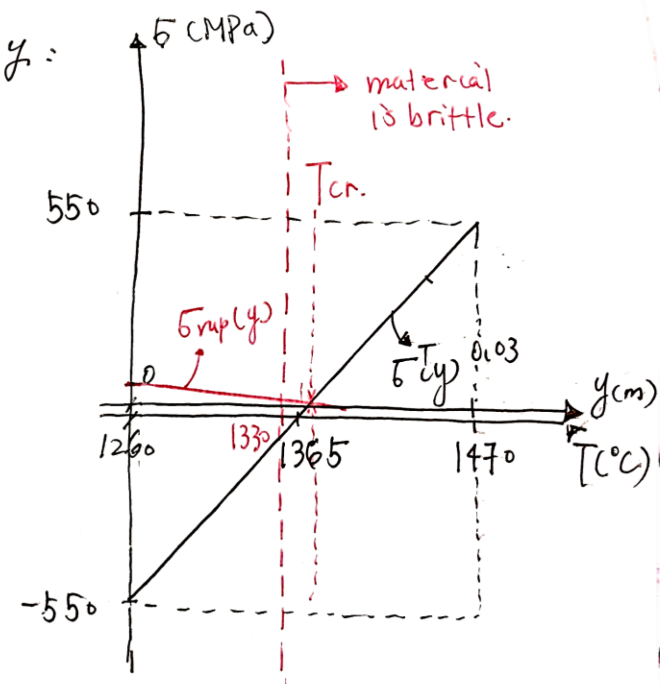
① The material is brittle, $T > T_{tr}$

T_{tr} = brittle-to-ductile transition temperature = 1330 °C (given)

(You can read about this effect in textbook page. 353 - 357)

② The thermal stress in the solid shell exceeds the strength of the shell (rupture stress)

$$\sigma_{rup}(T) = -0.125 \times T + 176 \text{ (MPa)}$$



— Stress distribution at instant B where reheating of surface of strand is the most severe.

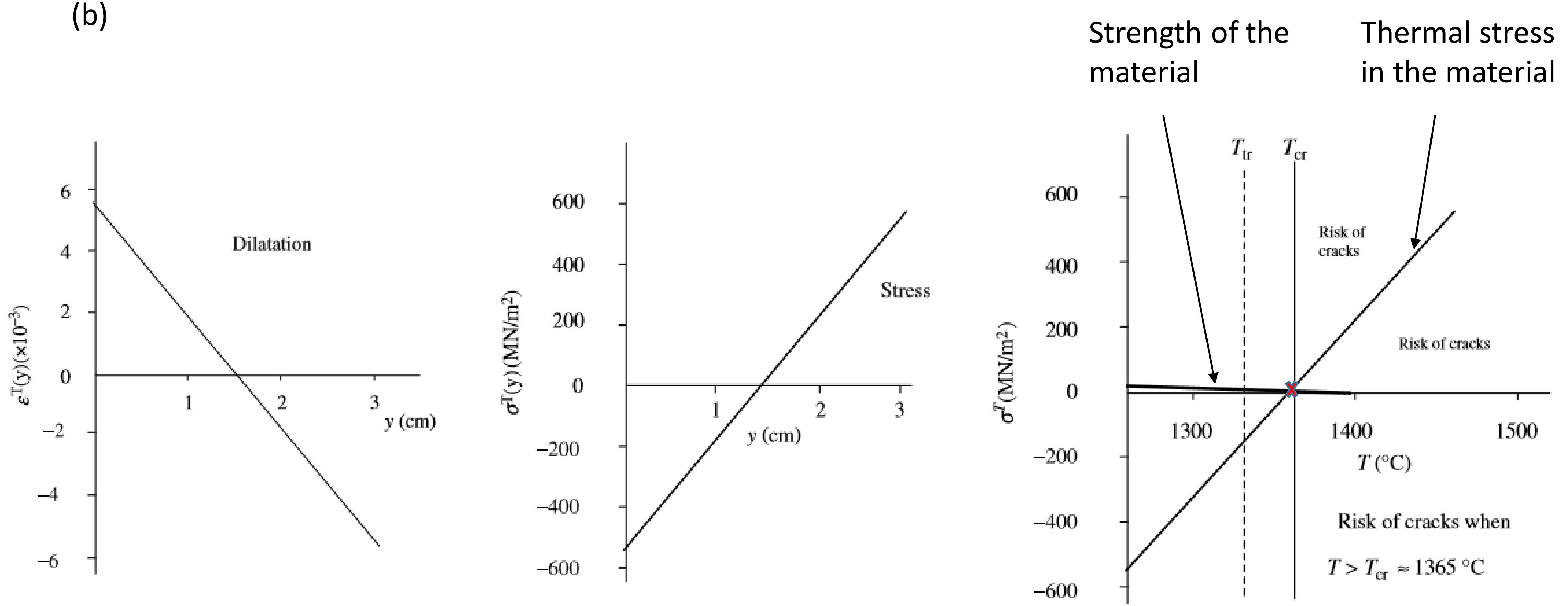
— Rupture stress limit of solid shell.

③ We identify the cross-over temperature $T_{cr} > 1330\,°C$ (T_{tr}) which indicates the thermal stress of the material starts to get higher than the rupture stress.

$$T_{cr} \approx 1366\,°C.$$

Hot cracks would happen at this temperature, $y \approx 15$ mm from surface of solid shell.

(b)



Strength of the material
Thermal stress in the material
Dilatation
$\varepsilon^T(y)(\times 10^{-3})$
y (cm)
Stress
$\sigma^T(y)$ (MN/m²)
y (cm)
T_{tr}
T_{cr}
Risk of cracks
Risk of cracks
σ^T(MN/m²)
T (°C)
Risk of cracks when
$T > T_{cr} \approx 1365$ °C

(10.10) (cont'd)

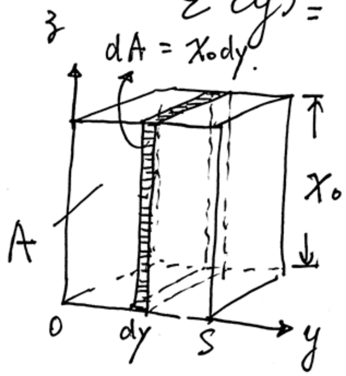
Derivation of thermal strain:

The thermal stress in a material acting on an internal area element is equal to the sum of "average stress" and the fictive fixed normal stress:

$$\varepsilon^T = \underbrace{\frac{-\iint_A \alpha \Delta T \, dA}{A}}_{\text{average stress}} + \underbrace{\alpha \Delta T}_{\text{fictive normal stress}}$$

consider $A = x_0 s$
$dA = x_0 dy$,

$$\varepsilon^T(y) = \frac{-\iint_A \alpha (T_A - T_B)(1-\frac{y}{s}) x_0 dy}{x_0 s} + \alpha (T_A - T_B)(1-\frac{y}{s})$$



T_A = Temperature at instant A at strand surface

T_B = Temperature at instant B at strand surface.

recall $\Delta T = (T_A - T_B)(1-\frac{y}{s})$. s = solidified thickness

$$\varepsilon^T(y) = \alpha (T_A - T_B)\left[\frac{-\int_0^s (1-\frac{y}{s}) x_0 dy}{x_0 \cdot s} + (1-\frac{y}{s})\right] = \alpha (T_A - T_B)(-\frac{1}{2} + 1 - \frac{y}{s})$$

$$= \alpha (T_A - T_B)(\frac{1}{2} - \frac{y}{s}) \quad \#$$