Problem 6.3

6.3 The dendrite arm distances in a cast material strongly influence the properties of the material. For an Al-base alloy, the dendrite arm distance varies with the solidification rate according to the following:

$$v_{\text{growth}}\,\lambda_{\text{den}}^2 = 1.0 \times 10^{-12}\,\text{m}^3/\text{s}$$

In a pressure casting process the solidification time is influenced by the pressure because the heat transfer number increases with increasing pressure. This can be described by $h = 400\,p$, where p is the pressure in atm and h is the heat transfer number measured in W/m^2 K. Calculate the dendrite arm distance as a function of the pressure. The temperature of the surroundings is 25 °C. The heat of fusion of the Al-base alloy is 398 kJ/kg. Other material constants are taken from standard tables.

6.3) Calculate the dendrite arm distance as a function of pressure, in a pressure casting process.

heat transfer increases with increasing pressure

(1) $h = 400 \cdot p$, p: pressure in atm

We are also given a function for dendrite arm distance λ_{den}

(2) $V_{growth} \cdot \lambda_{den}^2 = 1 \cdot 10^{-12}\ m^2/s$, V_{growth}: solidification rate

Set up a heat balance equation to connect p and λ_{den}.

$$\begin{pmatrix}\text{heat transfer}\\ \text{across interface}\end{pmatrix} = \begin{pmatrix}\text{Solidification}\\ \text{heat}\end{pmatrix}$$

$$hA(T_L - T_0) = \rho A(-\Delta H)$$

during time — dt — solidification front will move dy_L

(3) $h(T_L - T_0) = \rho(-\Delta H)\dfrac{dy_L}{dt}$, $\dfrac{dy_L}{dt}$ is the growth rate

Combine (2) and (3)

$$V_{growth} = \frac{dy_L}{dt} = \frac{10^{-12}}{\lambda_{den}^2} = \frac{h(T_L - T_0)}{\rho(-\Delta H)}$$

$$\lambda_{den} = \left(10^{-12} \cdot \frac{\rho(-\Delta H)}{h(T_L - T_0)}\right)^{1/2}, \text{ insert (1)}$$

$$\lambda_{den} = \left(10^{-12} \cdot \frac{\rho(-\Delta H)}{400p(T_L - T_0)}\right)^{1/2}$$

Given in text: $T_0 = 25\,°C$, $T_L = 660\,°C$

$-\Delta H = 398 \cdot 10^3\ J/kg$

$\rho_{Al} = 2.7 \cdot 10^3\ kg/m^3$

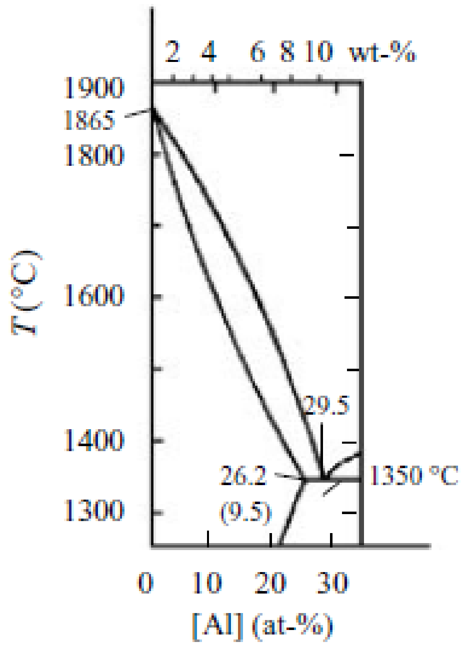
So $$\lambda_{den} = \left(10^{-12} \cdot \frac{2.7 \cdot 10^3 \cdot 398 \cdot 10^3}{400p\,(660 - 25)}\right)^{1/2}$$

$$\lambda_{den} = 6.5 \cdot 10^{-5} \cdot \frac{1}{\sqrt{p}}\ [m]$$

Problem 7.1

7.1 An Al–Zr alloy with 20 at-% Al is to be cast. Calculate the fraction of eutectic structure that is formed at 1350 °C.

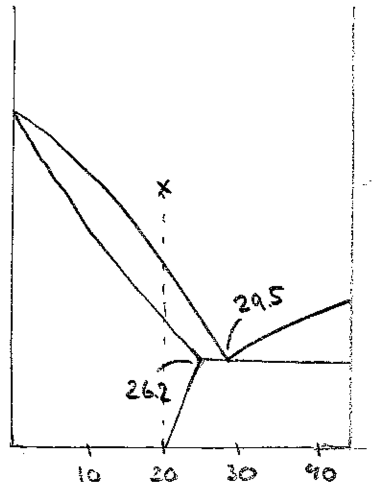
Hint B1



The phase diagram for the system Al–Zr is given.

7.1) Calculate the fraction of eutectic structure formed while casting an Al-Zr alloy.
The Al-Zr alloy have 20 at-% Al.
Due to microsegregation, homogenouos α-phase cannot form.
Look at the phase diagram

All liquid phase that is left when 1350°C is reached will form eutectic structure.



$$X^{0} = 20$$
$$X_E^L = 29.5$$
$$X_E^S = 26.2$$

Scheil's equation is valid

$$X_E^L = X_0^L (1-f_E)^{-(1-k_{part})} \quad \text{(at eutectic T)}$$

f_E - fraction of solid phase
remaining liquid at eutectic T is $(1-f_E)$

$$k_{part} = \frac{X_E^S}{X_E^L} = \frac{26.2}{29.5} = 0.89$$

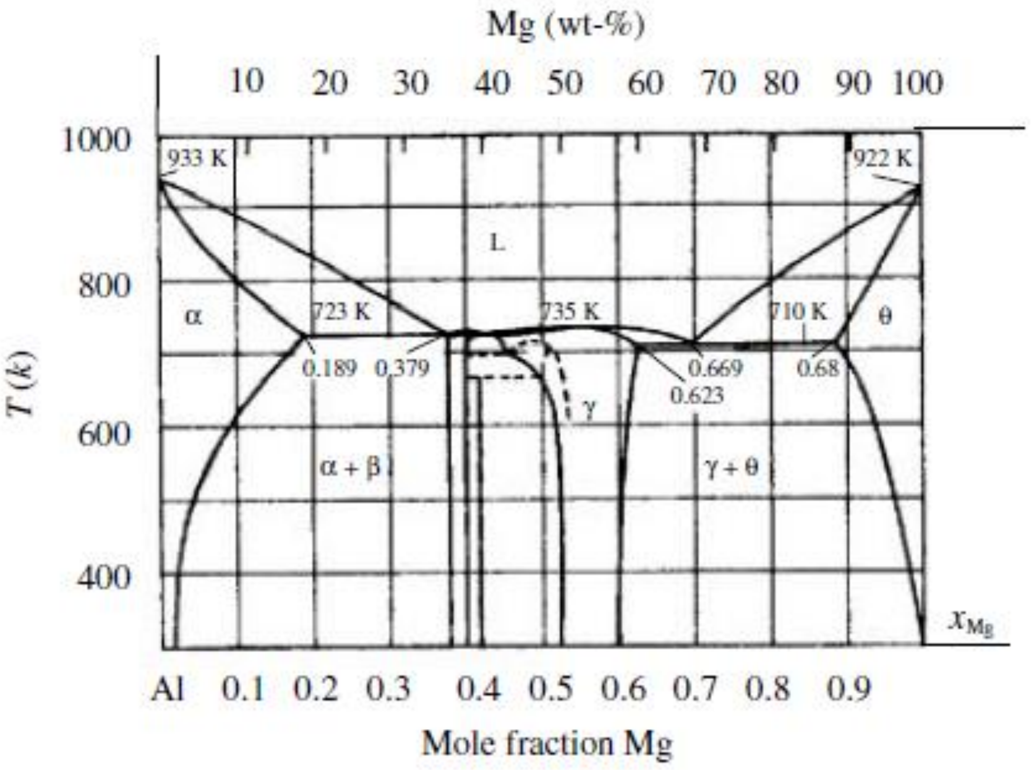
$$(1-f_E) = \left(\frac{X_0^L}{X_E^L}\right)^{\frac{1}{1-k_{part}}} = \left(\frac{20}{29.5}\right)^{\frac{1}{1-0.89}} = 0.029 = \underline{\underline{2.9\%}}$$

2.9 % eutectic structure is formed

Problem 7.2

7.2 An Al–Mg alloy with 40 at-% Al is to be cast. The circumstances are such that it is reasonable to assume that Scheil's equation is valid. The phase diagram of the system Al–Mg is given below.

Calculate the fraction of eutectic structure formed at solidification of the molten alloy.

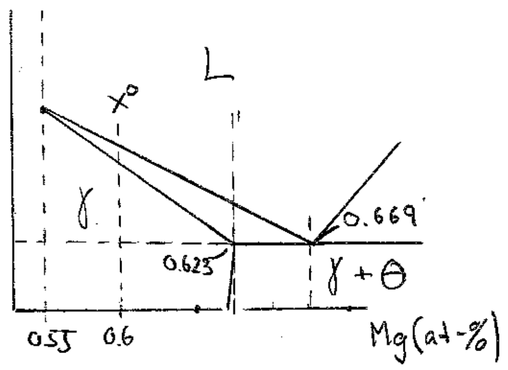


The values required for the calculations can be obtained from the text or read from the phase diagram.

7.2) An Al-Mg alloy with 40at-% Al is to be cast. Assume scheils equation is valid.
Calculate the fraction of eutectic structure formed at solidification.

Draw simplified phase diagram.

$L \rightarrow L + \gamma \rightarrow \gamma + \Theta$



Scheils equation at eutectic temperature

$$X_E^L = X_0^L \left(1 - f_E^S\right)^{-(1-k_{part})} \quad , \quad k_{part} = \frac{X_E^S}{X_E^L}$$

Remaining melt

$$f_E^L = \left(1 - f_E^S\right) = \left(\frac{X_0^L}{X_E^L}\right)^{\frac{1}{1-k_{part}}}$$

$X_0^L = 0.6$, $X_E^L = 0.669$, $X_E^S = 0.625$

$k_{part} = \frac{X_E^S}{X_E^L}$, but the L+$\gamma$ area starts at 0.55

$$k_{part} = \frac{X_E^S - 0.55}{X_E^L - 0.55} = \frac{0.623 - 0.55}{0.669 - 0.55} = 0.61$$

$f_E^L = \frac{X_0^L}{X_E^L}\left(\frac{1}{1-k_{part}}\right)$, again L + γ area doesn't ~~start~~ at 0

$$f_E^L = \frac{X_0^L - 0.55}{X_E^L - 0.55}\left(\frac{1}{1-k_{part}}\right) = \frac{0.6 - 0.55}{0.669 - 0.55}\left(\frac{1}{1-0.61}\right) = 0.108$$

11% of the structure formed during solidification will be eutectic.