In the cases when the solidification interval is broad, i. e. when the cooling power of the mould is poor and/or the thermal conductivity of the cast metal is high, the solidification process can be described in the way which is illustrated in figure 11. These conditions are valid for alloys with broad solidification intervals and for alloys, which solidify in sand moulds. We will come back to this in section 4.4.

The solidification process determines the properties of the cast material and is thus very important. Below we will treat the solidification process analytically and give examples on calculations of temperature distributions in the material and solidification times in case of ideal cooling.

4.3.2 Theory of Heat Transport at Casting with Ideal Contact between Metal and Mould

Solution of the Heat Equation

Theoretical calculations of the temperature and temperature changes as a function of position and time in a solidifying alloy melt after casting implies that it is necessary to find a solution of the general law of heat conduction (Fourier´s second law on page 8) in each special case.

$$\frac{\partial T}{\partial t} = \alpha \ \frac{\partial^2 T}{\partial y^2} \tag{10}$$

where α is the coefficient of thermal diffusion.

The solution of this partial differential equation of second order is the temperature T as a function of position y and time t. The solution contains two arbitrary constants, which are determined by use of given boundary conditions.

The Error Function

If a small amount of heat is distributed in an infinitely large body the solution of equation (10) will be

$$T = A_{\mathrm{o}} + \frac{B_{\mathrm{o}}}{\sqrt{t}} \cdot \mathrm{e}^{-\frac{y^2}{4\alpha t}} \qquad (15)$$

In many of the approximate solutions it is necessary to integrate the exponential function with respect to *y*. The integrated function is identical with the so-called *normal distribution function* or *error function*. It will be used in the following theoretical calculations of solidification processes and temperature distributions.

Normal Distribution Function

The normal distribution function appears in many connections and is often associated with problems of statistical nature for example error distribution. This is the reason why it also is called ”error function”, shortened ”erf”.

Definition: $$\mathrm{erf}\,(z) = \frac{2}{\sqrt{\pi}} \int_0^z \mathrm{e}^{-y^2}\,\mathrm{d}y \qquad (16)$$

To plot the error function, numerical values are required. They are listed in table 4.

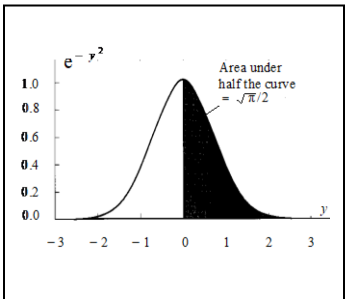


Figure 12.
Error function.

Table 4. The error function.

z	erf (z)	z	erf (z)	z	erf (z)	z	erf z
0.00	0.0000	0.40	0.4284	0.80	0.7421	1.40	0.9523
0.05	0.0564	0,45	0.4755	0.85	0.7707	1.50	0.9661
0.10	0.1125	0.50	0.5205	0.90	0.7969	1.60	0.9763
0.15	0.1680	0.55	0.5633	0.95	0.8209	1.70	0.9838
0.20	0.2227	0.60	0.6039	1.00	0.8427	1.80	0.9891
0.25	0.2763	0.65	0.6420	1.10	0.8802	1.90	0.9928
0.30	0.3286	0.70	0.6778	1.20	0.9103	2.00	0.9953
0.35	0.3794	0.75	0.7112	1.30	0.9340	∞	1.0000

The error function has the properties listed in table 5.

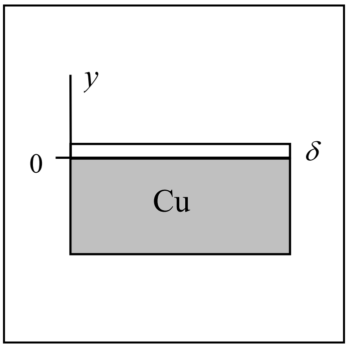
Table 5. Some properties of the error function.

1. erf (z) = the area under the curve within the interval $z = 0$ to $z = z$ (black area in figure 12).
2. $\dfrac{\mathrm{d\,erf}\,(z)}{\mathrm{d}z} = \dfrac{2}{\sqrt{\pi}}\,\mathrm{e}^{-z^2}$
3. $\mathrm{erf}\,(0) = 0$ and $\mathrm{erf}\,(\infty) = 1$
4. $\mathrm{erf}\,(-z) = -\,\mathrm{erf}\,(z)$ and $\mathrm{erf}\,(-\infty) = -1$
5. $\mathrm{erf}(z) = \dfrac{2}{\sqrt{\pi}} \cdot \left(z - \dfrac{z^3}{3} + \dfrac{z^5}{5} \ldots\ldots \right)$ for small values of z.

As a first example we choose the production of a thin metal film by rapid unidirectional cooling.

Example 2.

In order to achieve very high cooling rates of an alloy melt small metal droplets are shot towards a copper plate. In this way a very good contact between the copper plate and the melt is obtained. The melt is flattened out to a thin film of a thickness of a couple of hundreds µm.



The thickness of the metal layer is exaggerated in the figure, compared to the thickness of the copper plate.

Calculate the total solidification time, i.e. the time required for the melt to solidify completely, in terms of the following known data:

Casting temperature of the melt	$= T_{\mathrm{cast}}$
Solidus temperature of the melt	$= T_{\mathrm{S}}$
Liquidus temperature of the melt	$= T_{\mathrm{L}}$
Temperature of the Cu-plate	$= T_{\mathrm{o}}$
Thickness of the melt layer	$= \delta$
Heat capacitivity of the melt	$= c_{\mathrm{p}}$
Heat of fusion of the melt	$= -\Delta H$
Thermal diffusivity of the melt	$= \alpha$

Solution:

The heat transport through the layer is determined by the general heat equation, equation (10) on page 8

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{1′}$$

The solidification front (at the solidus temperature) is a horizontal plane, which moves upwards, starting at the Cu-plate. The solidification time is the time when the solidification front reaches the upper surface of the layer.

All thermal energy is transported towards the big Cu-plate. Because the layer is thin the Cu-plate receives only a small amount of heat. Under these circumstances the solution of equation (1′) is equal to equation (15) on page 18

$$T = A_o + \frac{B_o}{\sqrt{t}} \cdot e^{-\frac{y^2}{4\alpha t}} \tag{2′}$$

If the temperature distribution in the metal melt as a function of time is known we can calculate the time when the solidus temperature is achieved at the upper surface of the layer.

The first thing to do is to determine the constants A_o and B_o from the boundary conditions, which are valid in this case.

Boundary condition 1:

At the time $t = 0$ we have $T = T_o$ for $y = 0$ because the contact between the layer and the Cu-plate is very good. $T = T_o$ is also valid for all values of $y \neq 0$. We insert $T = T_o$ for $y \neq 0$ into equation (2′). The second term on the right-hand side becomes zero because the exponential term in the numerator approaches zero faster than the square root expression in the denominator and we get $A_o = T_o$.

Boundary condition 2:

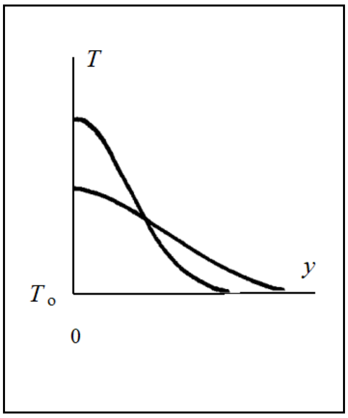
Two expressions of the total amount of heat, transferred to the Cu-plate, are found. They are equal, which gives the second boundary condition.

The whole melt layer cools from the casting temperature T_{cast} to the liquidus temperature T_L, solidifies completely and cools simultaneously from T_L to the solidus temperature T_S. The total amount of heat per unit area q, which is transferred to the Cu-plate, consists of cooling heat, when the temperature of the layer decreases from T_{cast} to T_{S}, and solidification heat

$$q = c_{\text{p}} \cdot \rho \cdot 1 \cdot \delta \cdot \left(T_{\text{cast}} - T_{\text{S}}\right) + \rho \cdot 1 \cdot \delta \left(-\Delta H\right) \qquad (3')$$

The amount of heat per unit area q must also be equal to the total ”excess heat” which is stored in the layer. We have to consider the fact that the temperature varies with the distance y from the Cu-plate and integrate the excess heat in many infinitesimal layers.

$$q = \int_0^{\delta} c_{\text{p}} \cdot \rho \cdot 1 \cdot \mathrm{d}y \cdot \left(T - T_{\text{o}}\right) \approx \int_0^{\infty} c_{\text{p}} \cdot \rho \cdot \left(\frac{B_{\text{o}}}{\sqrt{t}} \cdot \mathrm{e}^{-\frac{y^2}{4\alpha t}\mathrm{d}y} \right) \cdot \mathrm{d}y = c_{\text{p}}\, \rho\, B_{\text{o}} \cdot \int_0^{\infty} \frac{1}{\sqrt{t}} \cdot \mathrm{e}^{-\frac{y^2}{4\alpha t}\mathrm{d}y} \cdot \mathrm{d}y$$



Temperature distribution in the layer at two different times t_1 and t_2. The width of the layer is, for the sake of clearness, strongly enlarged.

Because $A_{\text{o}} = T_{\text{o}}$ we have $T - T_{\text{o}} = \frac{B_{\text{o}}}{\sqrt{t}} \cdot \mathrm{e}^{-\frac{y^2}{4\alpha t}}$, which has been inserted into the integral. It does not matter that the upper integral limit has been extended from δ to ∞ because $T = T_{\text{o}}$ within this interval and the contribution to the value of the integral will be zero.

After change of variable ($z = \frac{y}{\sqrt{4\alpha t}}$) and ($\mathrm{d}z = \frac{\mathrm{d}y}{\sqrt{4\alpha t}}$) we get

$$q = \frac{c_{\text{p}}\, \rho\, B_{\text{o}}}{\sqrt{t}} \cdot \int_0^{\infty} \mathrm{e}^{-\frac{y^2}{4\alpha t}} \cdot \mathrm{d}y = \frac{c_{\text{p}}\, \rho\, B_{\text{o}}}{\sqrt{t}} \cdot \int_0^{\infty} \mathrm{e}^{-z^2} \cdot \sqrt{4\alpha t} \cdot \mathrm{d}z \qquad (4')$$

The root expression can be moved in front of the integral sign and the value of the integral is obtained from figure 12 on page 18.

$$q = \frac{c_{\text{p}}\, \rho\, B_{\text{o}}}{\sqrt{t}} \cdot \sqrt{4\alpha t} \cdot \int_0^{\infty} \mathrm{e}^{-z^2} \cdot \mathrm{d}z = c_{\text{p}}\, \rho\, B_{\text{o}} \cdot \sqrt{4\alpha} \cdot \frac{\sqrt{\pi}}{2} = c_{\text{p}}\, \rho\, B_{\text{o}} \cdot \sqrt{\pi\alpha} \qquad (5')$$

If we combine equations (3′) and (5′) we get the value of the constant B_{o}:

$$B_{\text{o}} = \frac{\left[-\Delta H + c_{\text{p}}(T_{\text{cast}} - T_{\text{S}})\right] \cdot \delta}{c_{\text{p}} \sqrt{\pi\alpha}} \qquad (6')$$

Solution of the heat equation:

The determined values of B_o (equation (6′) and A_o ($A_o = T_o$) are inserted into equation (2′) and we get

$$T = T_o + \frac{\left[-\Delta H + c_p (T_{cast} - T_S)\right] \cdot \delta}{c_p \sqrt{\pi\alpha} \cdot \sqrt{t}} \cdot e^{-\frac{y^2}{4\alpha t}} \tag{7′}$$

The second solidification front of the alloy reaches the upper part of the metal layer at the very last time. We get the desired solidification time by inserting $y = \delta$ and $T = T_S$ in equation (7′). As the layer is very thin we can approximately set the exponent = 0. Then the exponential factor becomes = 1 and we get the answer by solving t.

Answer: The solidification time of the alloy will be

$$t = \left(\frac{-\Delta H + c_p (T_{cast} - T_S)}{c_p (T_S - T_o)} \right)^2 \cdot \frac{\delta^2}{\pi\alpha}$$

Temperature Distribution of a Metal Melt in a Metal Mould

Production of thin metal films by the method in example 2 is an unusual method. The normal situation is that both mould and metal melt are extended. The temperature distribution at the beginning of the solidification process is in principle illustrated in figure 13.

In this case the solution of Fourier′s second law (equation (10) on page 8) can be found by addition of a great number of sub-solutions like equation (15) on page 18 according to the superposition principle. Each infinitesimal layer dy contributes and we get

$$T - T_L = \int_{-\infty}^{\infty} \frac{B_o}{\sqrt{t}} \cdot e^{-\frac{y^2}{4\alpha t}} \cdot dy \tag{17}$$

This method will be applied to the solidification process of a metal melt in contact with a metal mould, based on the following assumptions:

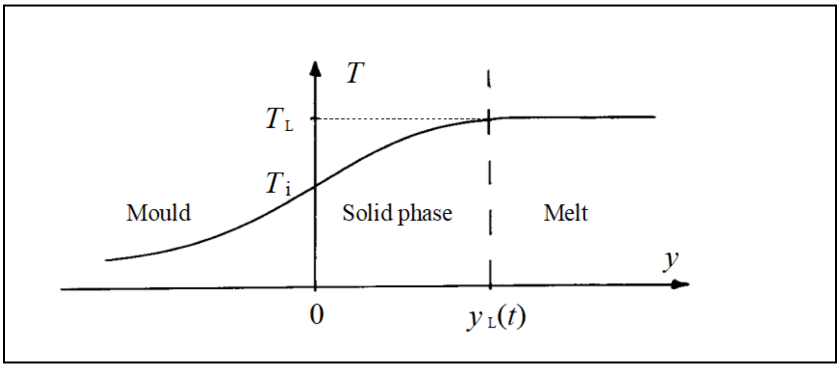


Figure 13.
Approximate temperature distribution in a solidifying but not superheated metal melt in a metal mould.

1. The contact between melt and mould is good, which means that there is no resistance against heat transport over the mould/metal interface.
2. The metal is not superheated.
3. The metal melt has a narrow solidification interval.
4. The volume of the mould is very large, approximately infinite.

The schematic temperature profile in figure 13 has been drawn according to these assumptions. The temperatures of metal and mould will adjust to equilibrium at the interface, where they are equal.

The temperature distribution in the metal and mould during solidification will be calculated as a function of y and t.

At the time $t = 0$ the metal is poured into the mould. The position of the y-axis is shown in figure 13. y is zero at the interface between metal and mould, positive in the metal and negative in the mould.

Since the metal and the mould are made of different materials with different properties, the differential equation (10) on page 8 must be solved separately for each of them. The symbols for the quantities used are listed on next page. The differential equations for the mould and the metal will then be:

$$\frac{\partial T_{\text{mould}}}{\partial t} = \alpha_{\text{mould}} \frac{\partial^2 T_{\text{mould}}}{\partial y^2} \qquad (18_{\text{mould}})$$

$$\frac{\partial T_{\text{metal}}}{\partial t} = \alpha_{\text{metal}} \frac{\partial^2 T_{\text{metal}}}{\partial y^2} \qquad (18_{\text{metal}})$$

Quantity	Mould	Metal
Temperature	$T_{mould}(t, y)$	$T_{metal}(t, y)$
Temperature at the interface	T_i	T_i
Temperature of the melt		T_L
Room temperature	T_o	
Coefficient of thermal diffusion	α_{mould}	α_{metal}
Coefficient of heat conduction	k_{mould}	k_{metal}

Solution of the General Equation of Thermal Conduction

The solutions to the equations (18) above have the same shape as equation (17) on page 22. Integration is necessary to add the contributions from all the dy layers (compare equation (17)). To find the solutions in this case we replace the variable y in equation (17) by $y = \sqrt{4\alpha t}$. At this variable transformation the factor $\sqrt{t}$ in the denominator in the second term disappears and is thus missing in the solutions of equations (18) (compare equations (4′) and (5′) on page 21). The erf function can be introduced in the solutions, which can be written

$$T_{mould} = A_{mould} + B_{mould} \cdot \mathrm{erf}\left(\frac{y}{\sqrt{4\alpha_{mould} t}}\right) \qquad (19_{mould})$$

$$T_{metal} = A_{metal} + B_{metal} \cdot \mathrm{erf}\left(\frac{y}{\sqrt{4\alpha_{metal} t}}\right) \qquad (19_{metal})$$

The four arbitrary constants in the solutions (19) will be determined by introducing *boundary conditions* valid for the present case.

The first ones are:

1. *Boundary Condition for the Mould*

$$T(t, -\infty) = T_o \qquad (20)$$

2. *Boundary Condition for the Interface Mould/Metal*

$$T(t,0) = T_{\mathrm{i}} \tag{21}$$

The relation $\frac{\partial q}{\partial t} = -k\frac{\partial T}{\partial y}$ (compare equation (3) on page 5) can be used to apply the law of energy conservation at the inter-face between mould and metal. The heat flux from the solid phase to the mould is equal to the heat flux absorbed by the lat-ter, which gives the third boundary condition:

3. *Boundary Condition for the Interface Mould/Metal*

$$k_{\mathrm{mould}}\frac{\partial T_{\mathrm{mould}}}{\partial y} = k_{\mathrm{metal}}\frac{\partial T_{\mathrm{metal}}}{\partial y} \tag{22}$$

At the solidification front the temperature is constant, i.e. inde-pendent of time, and equal to the liquidus temperature T_{L}. The solidification front moves, i.e. its position is a function of time. This can be described by the function

$$y = y_{\mathrm{L}}(t)$$

and we get the fourth boundary condition:

4. *Boundary Condition for the Solidification Front*

$$T(t, y_{\mathrm{L}}(t)) = T_{\mathrm{L}} \tag{23}$$

The solidified metal absorbs the heat of solidification, which is generated at the solidification front, because its temperature is lower than that of the melt. The velocity of the solidification front is equal to the derivative of $y_{\mathrm{L}}(t)$ with respect to time and the heat flux can be written

$$\frac{\mathrm{d}q}{\mathrm{d}t} = (-\Delta H)\cdot\rho\cdot 1\cdot\frac{\mathrm{d}y_{\mathrm{L}}(t)}{\mathrm{d}t}$$

and we get a fifth boundary condition:

5. *Boundary Condition for the Metal*

$$k_{\mathrm{metal}}\left(\frac{\partial T_{\mathrm{metal}}}{\partial y}\right)_{y=y_{\mathrm{L}}(t)} = (-\Delta H)\cdot\rho\cdot\frac{\mathrm{d}y_{\mathrm{L}}(t)}{\mathrm{d}t} \tag{24}$$

Equations (20), (21), (22), (23) and (24) are the set of boundary conditions. They will be used to determine the four constants in equations (19). There are four constants and five conditions. Thus the system seems to be overestimated. This is not the case, however, as it is necessary to determine a fifth constant, which will be introduced below.

At the solidification front we have (equation (19_{metal})

$$T_L = A_{\text{metal}} + B_{\text{metal}} \cdot \text{erf}\left(\frac{y_L}{\sqrt{4\alpha_{\text{metal}}\, t}}\right) \tag{25}$$

Since A_{metal} , B_{metal} and T_L are constants, the erf function must also be a constant, i. e. independent of t. The conclusion is that the variable of the function also must be constant which gives the condition:

$$y_L(t) = \lambda \cdot \sqrt{4\alpha_{\text{metal}} t} \tag{26}$$

λ is the fifth constant to be determined from the five boundary conditions. When λ *is* known the *growth rate* or *solidication rate* can easily be found by derivation of $y_L(t)$ with respect to t

$$\frac{\mathrm{d}y_L(t)}{\mathrm{d}t} = \lambda \cdot \sqrt{\frac{\alpha_{\text{metal}}}{t}} \tag{27}$$

Determination of the Constants from the Boundary Conditions

The expression for $y_L(t)$ in equation (26) is introduced in equation (25), which gives

$$T_L = A_{\text{metal}} + B_{\text{metal}} \cdot \text{erf}\ \lambda \tag{28}$$

Equations (20) is valid for the mould. It can be written

$$T_o = A_{\text{mould}} + B_{\text{mould}} \cdot \text{erf}\,(-\infty)$$

or, by use of property 4, page 19, we get

$$A_{\text{mould}} = T_o + B_{\text{mould}} \tag{29}$$

At the interface ($y = 0$) the temperature is equal for the metal and the mould (equation (21)) or $T_{\text{metal}} = T_{\text{mould}}$. We use this condition by inserting $y = 0$ into equations (19) and get

$$A_{\text{mould}} + B_{\text{mould}} \cdot 0 = A_{\text{metal}} + B_{\text{metal}} \cdot 0$$

or, by use of equation (29), we get

$$A_{\text{mould}} = A_{\text{metal}} = T_{\text{o}} + B_{\text{mould}} \tag{30}$$

The expression for A_{metal} (equation (30)) is introduced into equation (28), which gives

$$T_{\text{L}} = T_{\text{o}} + B_{\text{mould}} + B_{\text{metal}} \cdot \operatorname{erf} \lambda$$

or

$$B_{\text{mould}} = T_{\text{L}} - T_{\text{o}} - B_{\text{metal}} \cdot \operatorname{erf} \lambda \tag{31}$$

To be able to use the boundary condition in equation (22) we have to derive the equations (19) with respect to y. We also use property 2 of the erf function (see page 19)

$$\frac{\partial T_{\text{mould}}}{\partial y} = B_{\text{mould}} \cdot \frac{2}{\sqrt{\pi}} \cdot \mathrm{e}^{-\frac{y^2}{4\alpha_{\text{mould}} t}} \cdot \frac{1}{\sqrt{4\alpha_{\text{mould}}\, t}}$$

and

$$\frac{\partial T_{\text{metal}}}{\partial y} = B_{\text{metal}} \cdot \frac{2}{\sqrt{\pi}} \cdot \mathrm{e}^{-\frac{y^2}{4\alpha_{\text{metal}} t}} \cdot \frac{1}{\sqrt{4\alpha_{\text{metal}}\, t}}$$

These expressions of the derivatives are introduced into equation (22) and we get for $y = 0$

$$k_{\text{mould}} B_{\text{mould}} \cdot \frac{1}{\sqrt{4\,\alpha_{\text{mould}}\, t}} = k_{\text{metal}} B_{\text{metal}} \cdot \frac{1}{\sqrt{4\,\alpha_{\text{metal}}\, t}}$$

which can be rewritten as

$$B_{\text{mould}} = B_{\text{metal}} \cdot \frac{k_{\text{metal}} \sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}} \sqrt{\alpha_{\text{metal}}}} \tag{32}$$

This expression of B_{mould} is introduced in equation (31), which is solved for B_{metal}

$$B_{\text{metal}} = \frac{T_{\text{L}} - T_{\text{o}}}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} + \operatorname{erf}\lambda} \qquad (33)$$

By combining equations (32) and (33) we get

$$B_{\text{mould}} = \frac{T_{\text{L}} - T_{\text{o}}}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} + \operatorname{erf}\lambda} \cdot \frac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} \qquad (34)$$

Equation (30) in combination with equation (34) gives

$$A_{\text{mould}} = A_{\text{metal}} = T_{\text{o}} + \frac{T_{\text{L}} - T_{\text{o}}}{\dfrac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} + \operatorname{erf}\lambda} \cdot \frac{k_{\text{metal}}\sqrt{\alpha_{\text{mould}}}}{k_{\text{mould}}\sqrt{\alpha_{\text{metal}}}} \qquad (35)$$

Now we have the four constants expressed in known quantities and the unknown constant λ [equations (33, (34) and (35)]. It can be determined from the last boundary condition, equation (24) in combination with equation (26). The velocity of the solidification front is determined by the temperature gradient in the solid metal.

The values of A_{metal} and B_{metal} are introduced in equation (19_{metal}) and we derive the new equation with respect to *y*. Equation (26) is derived with respect to *t* and the two derivatives obtained are inserted into equation (24). The final result is

$$\frac{c_{\text{p}}^{\text{metal}}(T_{\text{L}} - T_{\text{o}})}{-\Delta H} = \sqrt{\pi} \cdot \lambda\, e^{\lambda^2} \cdot \left(\sqrt{\frac{k_{\text{metal}}\rho_{\text{metal}}c_{\text{p}}^{\text{metal}}}{k_{\text{mould}}\rho_{\text{mould}}c_{\text{p}}^{\text{mould}}}} + \operatorname{erf}\lambda \right) \qquad (36)$$

In equation (36) the relation $\alpha = \dfrac{k}{\rho\, c_{\text{p}}}$ (equation (11) on page 8) has been used.

From equation (36) it is possible to solve the constant λ. It is not possible to find an exact analytical solution to the equation. It can be solved by iteration and use of a calculator or graphically.

The curves in figures (14) and (15) may be used to find a reasonable introductory value of λ for iteration.

The solution procedure is illustrated in example 3.

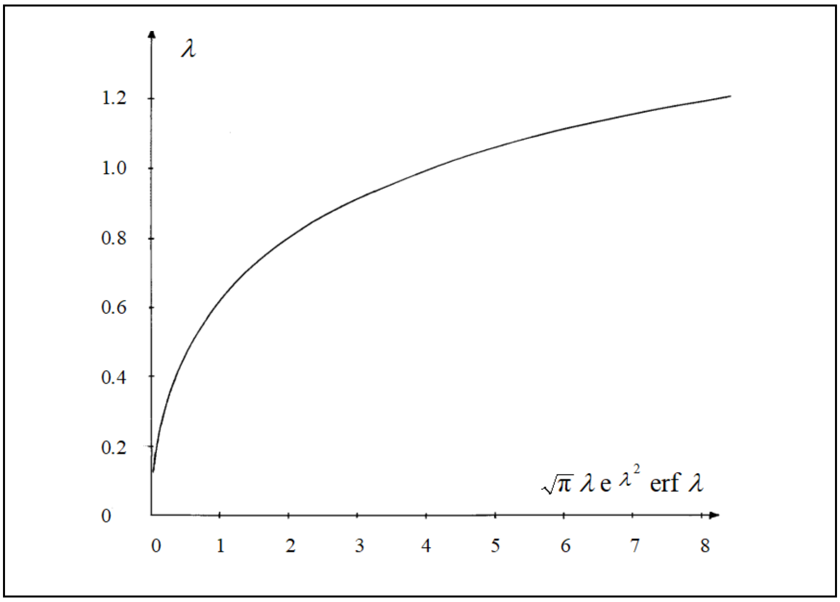


Figure 14.

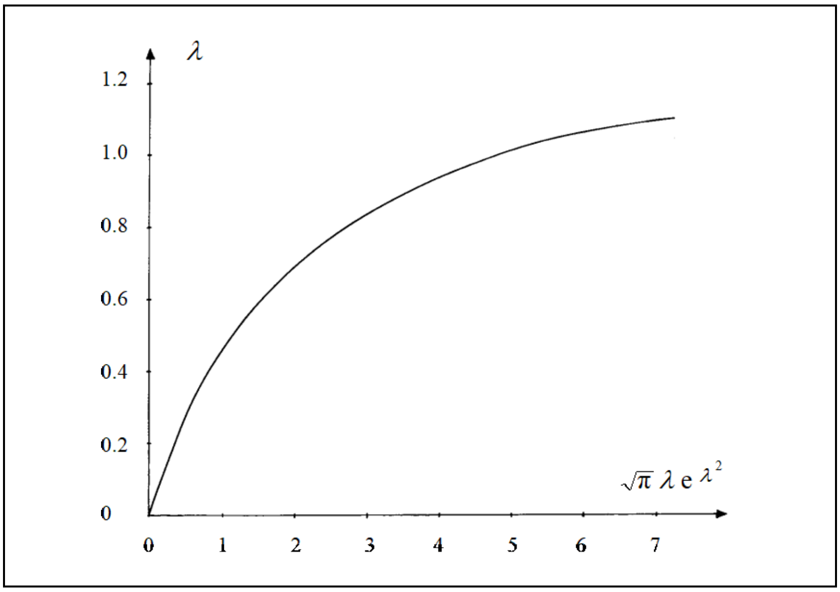


Figure 15.

Example 3.

The equation below is valid at casting in case of good contact between mould and metal

$$\frac{c_p^{\text{metal}}(T_L - T_o)}{-\Delta H} = \sqrt{\pi} \cdot \lambda\, e^{\lambda^2} \cdot \left(\sqrt{\frac{k_{\text{metal}} \rho_{\text{metal}} c_p^{\text{metal}}}{k_{\text{mould}} \rho_{\text{mould}} c_p^{\text{mould}}}} + \text{erf}\, \lambda \right)$$

The constant, λ, which can be calculated from this equation, is used to describe the position of the solidification front as function of time ($y_L(t) = \lambda \cdot \sqrt{4\alpha_{\text{metal}}\, t}$).

Calculate λ by iteration for the special casting process, which corresponds to the equation

$$3.20 = \sqrt{\pi} \cdot \lambda\, e^{\lambda^2} \cdot (0.53 + \text{erf}\, \lambda) \qquad (1')$$

Solution:
We guess an initial value of, λ, for example $\lambda = 0.6$, and test it roughly by aid of figures 14 and 15 on page 29.

For $\lambda = 0.6$ we read the value $\sqrt{\pi} \cdot \lambda\, e^{\lambda^2} \approx 1.6$ in figure 15.
For $\lambda = 0.6$ we read the value $\sqrt{\pi} \cdot \lambda\, e^{\lambda^2} \cdot \text{erf}\,(\lambda) \approx 1.0$ in figure 14.

Inserting these values into equation (1′) we get

$$\sqrt{\pi} \cdot \lambda\, e^{\lambda^2} \cdot (0.53 + \text{erf}\, \lambda) = 1.6 \cdot 0.53 + 1.0 = 1.8$$

This value is obviously too low (it should be 3.20) but it is of the right magnitude. Next, we make a table and calculate the values by aid of a calcu-lator and table 4 on page 18.

λ	erf λ (from table 4)	0.53 + erf λ	$\sqrt{\pi} \cdot \lambda\, e^{\lambda^2}$	$\sqrt{\pi} \cdot \lambda\, e^{\lambda^2} \cdot (0.53 + \text{erf}\, \lambda)$
0.60	0.6039	1.1339	1.5243	1.73
0.70	0.6778	1.2078	2.0252	2.45
0.75	0.7112	1.2412	2.3331	2.90
0.80	0.7421	1.2721	2.6891	3.42

Interpolation within the interval $0.75 < \lambda < 0.80$ gives the final value.
Answer: $\lambda = 0.78$.---
