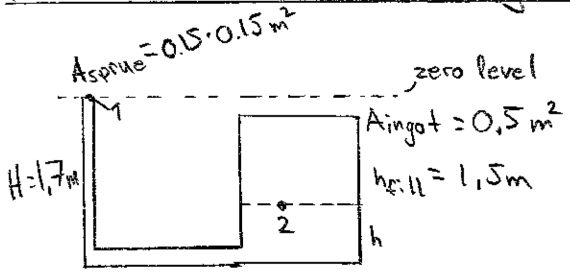
3.3 Calculate the filling time, t_{fill}!



If we know v_2 we can find t_{fill}!

(3) $v_2 = \frac{dh}{dt} = v_{ingot}$

And we get v_2 from the continuity principle. (incompressible fluid)

(4) $A_{sprue} \cdot v_{sprue} = A_{ingot} \cdot v_{ingot}$

In this case 6 ingots.

(3)(4): $A_{sprue} \cdot v_{sprue} = 6 A_{ingot} \cdot \frac{dh}{dt}$

(5) $v_{sprue} = \frac{6 A_{ingot}}{A_{sprue}} \cdot \frac{dh}{dt} = v_1$

We have lamier flow, so use Bernoulli's eq.

$$p_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = p_2 + \rho g h_2 + \frac{\rho v_2^2}{2} \quad (1)$$

$p_1 = p_2 = p_{atm}$, $h_1 = 0$, $h_2 = H - h$ $\Rightarrow$ $p_{atm} + 0 + \frac{\rho v_1^2}{2} = p_{atm} + \rho g (H-h) + \frac{\rho v_2^2}{2}$

$$v_1^2 = 2g(H-h) + v_2^2$$

(How do we find the filling time?)

Since $v_1 >> v_2$ we can neglect v_2 $\rightarrow$ $v_1 = \sqrt{2g(H-h)}$ (2)

introduce (5) into (2)

$$\frac{6 A_{ingot}}{A_{sprue}} \cdot \frac{dh}{dt} = \sqrt{2g(H-h)}$$

Separate the variables ... and integrate

$$dt = \frac{6 A_{ingot}}{\sqrt{2g}\, A_{sprue}} \cdot \frac{dh}{\sqrt{H-h}}, \quad \int_0^t dt = \frac{6 A_{ingot}}{\sqrt{2g}\, A_{sprue}} \int_0^h \frac{dh}{\sqrt{H-h}}$$

$$t = \frac{6 A_{ingot}}{\sqrt{2g}\, A_{sprue}} \cdot 2\left(\sqrt{H} - \sqrt{H-h}\right)$$

insert $t = t_{fill}$, $h = h_{fill}$

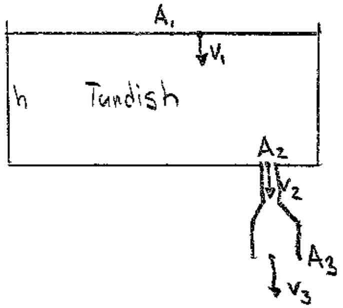
$$t_{fill} = \frac{6 \cdot 0.5}{\sqrt{2 \cdot 9.81} \cdot 0.15^2} \cdot 2\left(\sqrt{1.7} - \sqrt{1.7 - 1.5}\right) = \underline{\underline{51\ s}}$$

(Neglecting v_2 in bernoulli.

$v_1^2 - v_2^2 = 2g(H-h)$

$\left(\frac{dh}{dt} \cdot \frac{6 A_2}{A_1}\right)^2 - \left(\frac{dh}{dt}\right)^2 \Rightarrow \sqrt{\left(\frac{6 A_2}{A_1}\right)^2 - 1} \cdot \frac{dh}{dt} = \sqrt{2g(H-h)}$)

3.4 Calculate the casting rate?



Casting rate depends on outlet velocity, v_2, which in turn depends on the height of the melt.

We assume laminar flow and use Bernoulli's eq.

$$P_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g h_2 + \frac{\rho v_2^2}{2}$$

We can see that $A_1 >> A_2$ so $V_2 >> V_1 \approx 0$, also $P_1 = P_2 = P_{atm}$

Use outlet level as zero, $h_2 = 0$, $h_1 = h$ ($V_1 = 0$, because tundish level is stationary during casting.)

$$P_{atm} + \rho g h + 0 = P_{atm} + 0 + \frac{\rho v_2^2}{2} \implies V_2 = \sqrt{2gh}$$

How do we find the casting rate? V_3!

Continuity principle: (eliminate v_2)

$$A_2 v_2 = A_3 v_3 \longrightarrow V_2 = \frac{A_3}{A_2} \cdot V_3$$

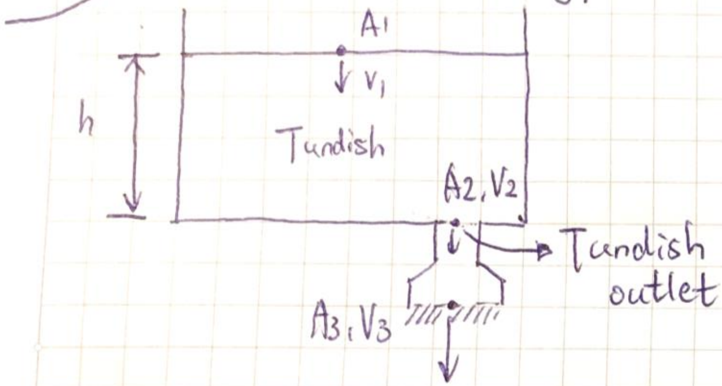
we get that

$$V_3 = \frac{A_2}{A_3} \cdot \sqrt{2gh} = \frac{\pi \cdot 0.005^2}{0.140^2} \cdot \sqrt{2 \cdot 9.81} = 0.018 \cdot \sqrt{h}$$

(The book forgot this 2) $V_3 = 0.010 \cdot \sqrt{h}$

3.5) Calculate casting rate and outlet diameter of tundish.

Same analogy as exercise 3.4. (Ignore the line "a machine with two lines", treat it as single tundish, single outlet")



Recall from 3.4:

$$p_1 + \rho g h_1 + \frac{\rho V_1^2}{2} = p_2 + \rho g h_2 + \frac{\rho V_2^2}{2}$$

Let $p_1 = p_2 = p_{atm}$, $h_2 = 0$ (zero potential)
$h_1 = h$.

$$\Rightarrow \rho g h + \frac{\rho V_1^2}{2} = \frac{\rho V_2^2}{2}.$$

Since $A_1 \gg A_2$, $V_1 \ll V_2$, ignore V_1

$$\Rightarrow V_2 = \sqrt{2gh}.$$

Melting capacity $= 70 \cdot 10^3$ kg/hr $=$ (Casting rate) $\cdot$ (ρ_{steel}) $\cdot$ (A_{slab})

Hence $V_{cast} = V_3 = \frac{70 \cdot 10^3}{(7200) \cdot (0.2 \times 1.5)} = 32.4$ m/hr $= 9 \cdot 10^{-3}$ m/sec.

Use continuity equation $A_3 V_3 = A_2 V_2$, and $V_2 = \sqrt{2gh}$, $h = 0.5$ m.

We have $A_2 = \frac{\pi (D_2)^2}{4} = \frac{A_3 V_3}{V_2} = \frac{(0.2 \times 1.5)(9 \cdot 10^{-3})}{\sqrt{2 \cdot 9.81 \cdot 0.5}} = 8.62 \times 10^{-4}$ m^2

$$\therefore D_2 = \sqrt{\frac{(8.62 \cdot 10^{-4}) 4}{\pi}} = 0.033 \text{ m}$$

$= 3.3$ cm

Problem 3.7

- Cooling capacity of the mold

  A large cooling capacity means that heat is transported away quickly and the melt will solidify quickly. Results in low L_f.

- Surface tension

  High surface tension lowers the fluidity of the melt.

- Viscosity of the melt

  Incresed viscosity means flow fluidity. Higher melt temperature lowers the viscosity.

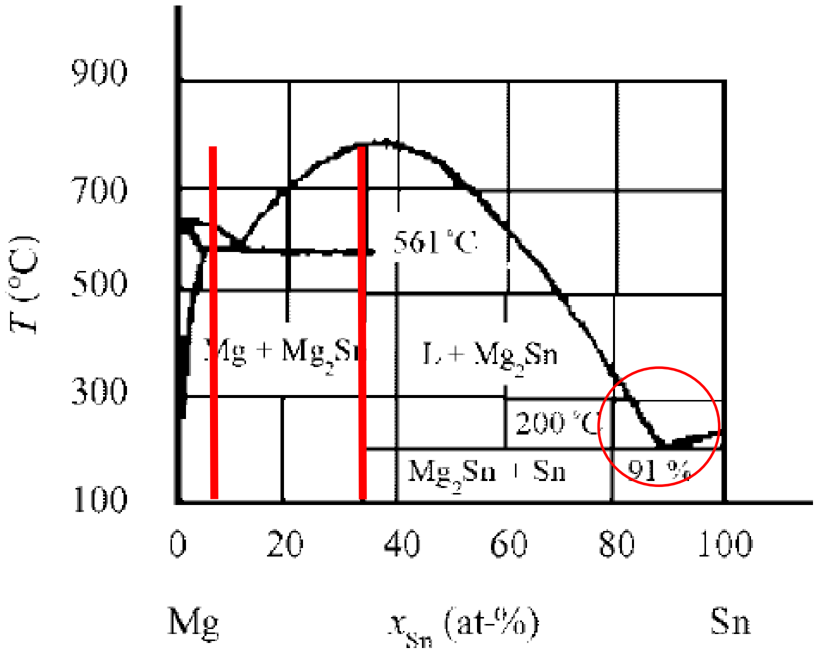
$$\eta = \eta_0 \exp\left(-\frac{Q_\eta}{RT}\right)$$

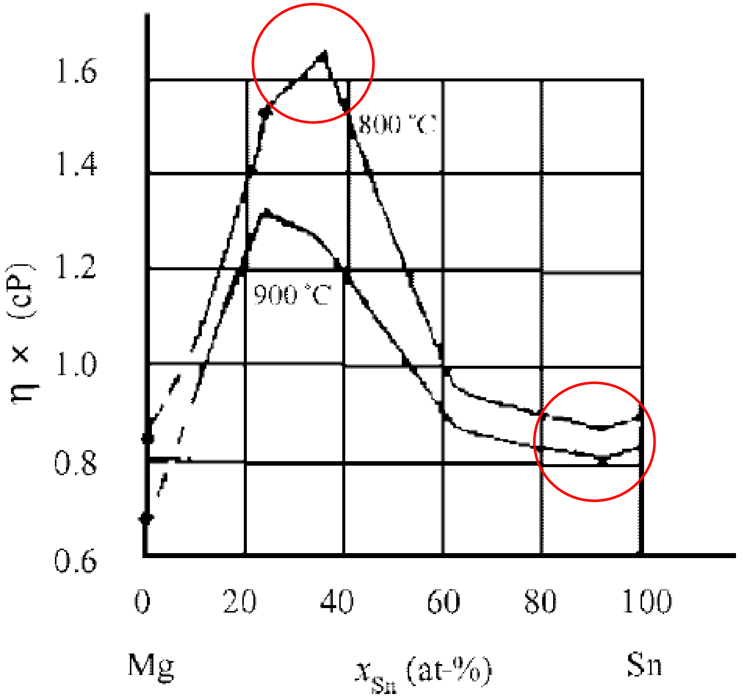
Problem 3.7 (cont'd)

- Composition of the melt

  Pure metals and eutectic alloys have low viscosity and therefore higher L_f. Intermediate phases often have viscosity maxima due to strong interatomic forces and thus a lower L_f.

  Small additions of impurities widens the solidification interval which lowers L_f.



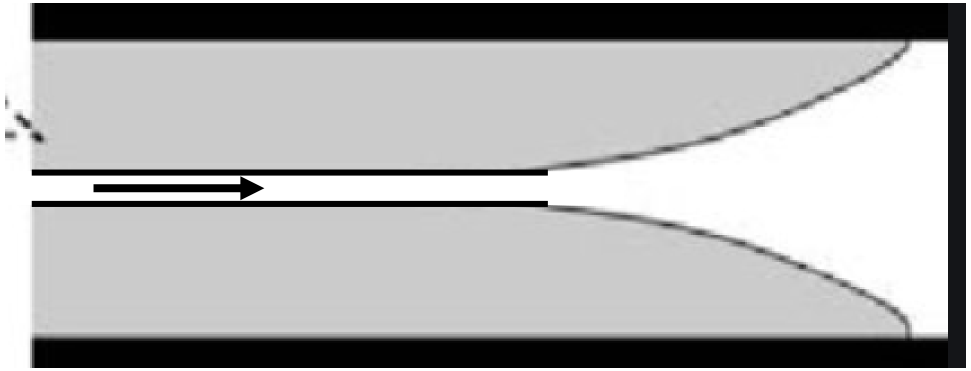


Problem 3.7 (cont'd)

Other factors?

- Solidification mode

  At 50% solid fraction, dendrites stops the melt from moving forward due to networking dendrites. In pure metals and eutectic alloy, the solidification front is planar. It starts at the mold wall and gradually progresses inwards. At 50% solid fraction it does not block the melt flow as the case in dendritic solidification.



Planar growth front

Dendritic growth front

C.M. GOURLAY, K. NOGITA, J. READ, and A.K. DAHLE. Journal of ELECTRONIC MATERIALS, Vol. 39, No. 1, 2010

Problem 3.7 (cont'd)

Other factors?

- Latent heat of fusion

  Higher latent heat of fusion increases L_f.

- Thermal conductivity of melt

  Lower thermal conductivity of melt reduces the rate heat is being carried away, increases L_f.

- Flow rate of the melt

  Laminar flow gives greater fluidity length than turbulent flow.