4.3.3 Theory of Heat Transport at Casting with Poor Contact between Metal and Mould. Solidification Rate. Solidification Time

When the contact at the interface between mould and metal is good and the melt is not superheated we get the temperature distribution illustrated in figure (13) on page 23. The temperatures in the mould and the metal are equal at the interface.

At poor contact between the mould and the metal there is a discontinuity of the temperature at the interface.

Figure 16 shows the temperature in both the metal and the mould some time after the casting. There are two temperatures at the interface, one in the metal, $T_{i\ metal}$, and one in the mould $T_{i\ mould}$. Both temperatures vary with time. T_o is the temperature of the surroundings of the mould.

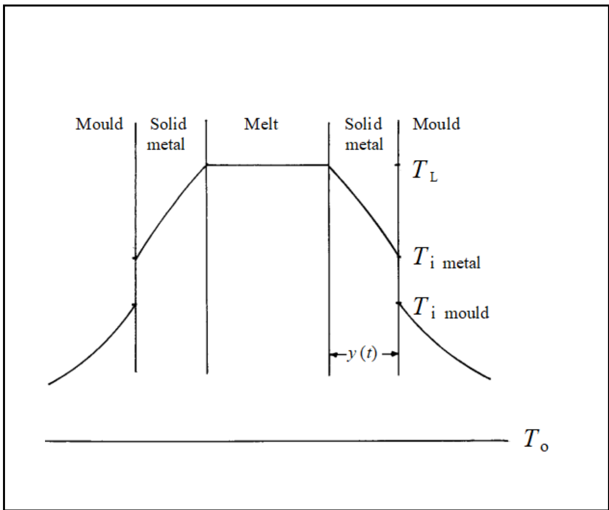

Figure 16.
Temperature distribution in mould and metal at poor contact at the interface at the time *t* after the casting.

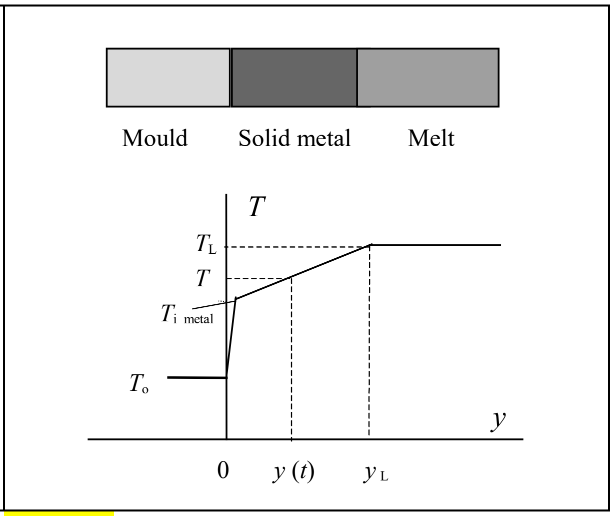

Figure 17.
Temperature distribution in the metal during the solidification process after the casting.

Figure 17 shows another example. An effective heat transport in the mould is assumed. Either the thermal conductivity of the mould has to be very good or the outer surface of the mould has to be directly cooled with water or air. In the latter case the heat transport mechanism is convection. T_o is the temperature of the surroundings of the mould.

Solidification Rate

The solidification process indicated in figure 17 is very common. The melt solidifies first at the interface between the metal and the mould. Initially the thin metal shell is kept in good contact with the mould by the pressure from the melt.

After some time the solidifying metal shell has become thick enough to resist the pressure from the melt. At the same time it cools and shrinks and suddenly looses the contact with the mould wall. At that moment the heat conduction decreases drastically.

We will analyse this solidification process, i. e. we want the temperature as a function of position and time and derive an expression of the solidification time.

The temperature is a function of two variables, the position y and the time t. Thus we must use partial derivatives.

The amount of heat per unit area, which passes a cross-section area perpendicular to the heat flux at the interface between the mould and the solid metal (see figure 17), can be written

$$-\frac{\partial q}{\partial t} = \rho(-\Delta H)\cdot\frac{\mathrm{d}y_{\mathrm{L}}(t)}{\mathrm{d}t} + \rho\, c_{\mathrm{p}} \int_0^{y_{\mathrm{L}}(t)} -\frac{\partial T}{\partial t}\cdot \mathrm{d}y \qquad (37)$$

The term on the left-hand side is the heat flux *lost* to the surroundings. For this reason a minus sign has to be added.

The *first* term on the right-hand side describes the amount of solidification heat per unit area and unit time released at position y_{L} when the solidification front moves with the rate $\mathrm{d}y_{\mathrm{L}}(t)/\mathrm{d}t$. The heat is transported away by means of heat conduction through the interface solid/mould.

The *second* term describes the amount of cooling heat in the solid per unit area and unit time, which is transported away by aid of heat conduction through the interface solid/mould.

Unfortunately $(-\Delta H)$ and c_{p} are not constant for all metals and all casting processes. This matter will be discussed in section 4.3.4 on page 38. In equation (37) above and in the following calculations below we have assumed that they are constants.

The total heat conduction through the interface solid/mould is described by the general law

$$\frac{\partial q}{\partial t} = -k\frac{\partial T}{\partial y} \quad \text{within the interval } 0 < y < y_L \tag{38}$$

This expression is introduced into equation (37), which gives

$$k\frac{\partial T}{\partial y} = \rho\left(-\Delta H\right)\cdot\frac{\mathrm{d}y_L(t)}{\mathrm{d}t} + \rho c_p \int_0^{y_L(t)} -\frac{\partial T}{\partial t}\,\mathrm{d}y \tag{39}$$

The differential dT is a function of both dy and dt.

$$\mathrm{d}T = \frac{\partial T}{\partial y}\cdot\mathrm{d}y + \frac{\partial T}{\partial t}\cdot\mathrm{d}t \tag{40}$$

If $\partial T/\partial t$ is small the second term in equations (37), (39) and (40) can be neglected and the rest of equation (39) can be integrated. The partial derivative $\partial T/\partial y$ is replaced by dT/dy. In or-der to find a simple approximate solution of equation (39) we initially assume that $\partial q/\partial t$ is constant and independent of y. As a consequence of this assumption and equation (37) we conclude that d$y_L(t)$/dt is also approximately independent of y. If these assumptions are valid we get

$$\int_{T_{\mathrm{i\,metal}}}^{T_L} k\,\mathrm{d}T = \int_0^{y_L(t)} \rho\left(-\Delta H\right)\cdot\frac{\mathrm{d}y_L(t)}{\mathrm{d}t}\,\mathrm{d}y \tag{41}$$

Provided that the heat of solidification ($-\Delta H$) is constant we get

$$k\left(T_L - T_{\mathrm{i\;metal}}\right) = \rho\left(-\Delta H\right)\cdot\frac{\mathrm{d}y_L(t)}{\mathrm{d}t}\,y_L(t) \tag{42}$$

In equation (42) the temperature T_L is constant. To be able to calculate the solidification rate, i.e. the velocity of the solidification front, d$y_L(t)$/dt, we have to know the amount of heat transported across the interface per unit time and unit area or the heat flux dq/dt. This heat flux is described by equation (5) on page 6, which can be applied here.

$$\frac{\mathrm{d}q}{\mathrm{d}t} = h\left(T_{\mathrm{i\;metal}} - T_o\right) \tag{43}$$

where h is the coefficient of heat transfer. It depends on many quantities and is hard to calculate theoretically. It is normally determined by experimental methods.

We have two expressions of the heat flux given in equations (38) and (43). If we replace the derivative in equation (38) by a linear function we get

$$h\left(T_{\text{i metal}} - T_{\text{o}}\right) = k \cdot \frac{T_{\text{L}} - T_{\text{i metal}}}{y_{\text{L}}(t)} \tag{44}$$

$T_{\text{i metal}}$ can be solved from equation (44):

$$T_{\text{i metal}} = \frac{T_{\text{L}} - T_{\text{o}}}{1 + \frac{h}{k} y_{\text{L}}(t)} + T_{\text{o}} \tag{45}$$

By introducing this expression of $T_{\text{i metal}}$ into equation (42) and combining it with equation (44) we get the velocity of the solidification front

$$\frac{\mathrm{d}y_{\text{L}}}{\mathrm{d}t} = \frac{T_{\text{L}} - T_{\text{o}}}{\rho(-\Delta H)} \cdot \frac{h}{1 + \frac{h}{k} \cdot y_{\text{L}}} \tag{46}$$

This is the desired expression of the solidification rate, provided that $(-\Delta H)$ is constant.

Solidification Time

To find the solidification time we transform equation (46) by separating the variables and integrating it.

$$\int_0^t \mathrm{d}t = \int_0^{y_{\text{L}}} \frac{\rho(-\Delta H)}{h\left(T_{\text{L}} - T_{\text{o}}\right)} \cdot \left(1 + \frac{h}{k} \cdot y_{\text{L}}\right) \mathrm{d}y_{\text{L}} \tag{47}$$

which gives the time it takes to achieve a shell of thickness y_{L}:

$$t = \frac{\rho(-\Delta H)}{T_{\text{L}} - T_{\text{o}}} \cdot \frac{y_{\text{L}}}{h} \cdot \left(1 + \frac{h}{2k} \cdot y_{\text{L}}\right) \tag{48}$$

Equation (48) can be used to calculate the total solidification time when the dimensions of the metal melt are known and $(-\Delta H)$ is constant.