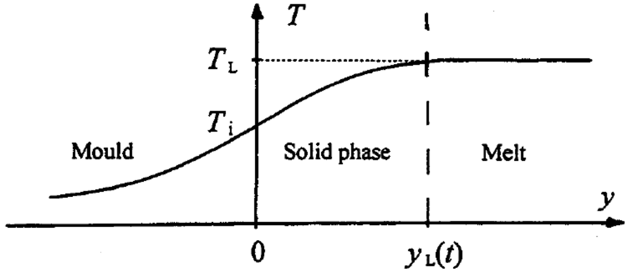
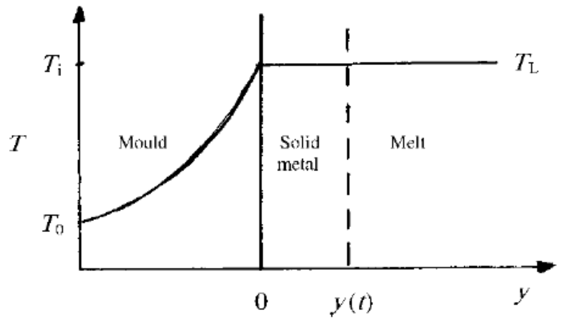
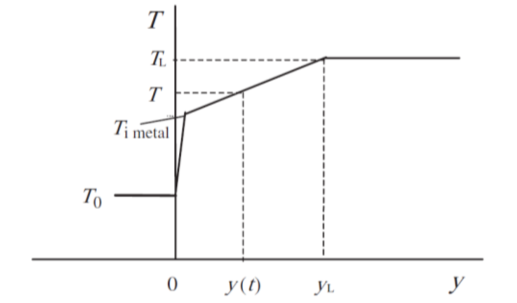
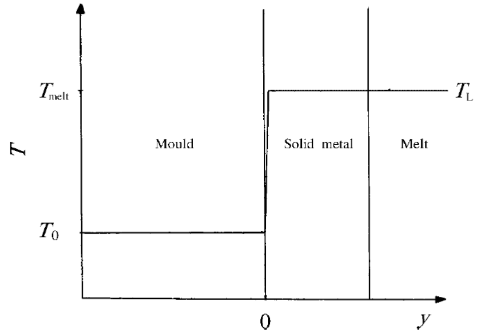
<table>
<tr>
<td rowspan="2">Ideal contact between mould and metal</td>
<td>General case

$$y_L(t) = \lambda \cdot \sqrt{4\alpha_{metal} t}$$
$$\frac{c_p^{metal}(T_L - T_0)}{-\Delta H} = \sqrt{\pi}\,\lambda \exp^{(\lambda^2)}\left(\sqrt{\frac{k_{metal}\rho_{metal}c_p^{metal}}{k_{mould}\rho_{mould}c_p^{mould}}} + \text{erf}\ \lambda\right)$$</td>
<td>
</td>
</tr>
<tr>
<td>Special case: Poor conductivity of mould: sand casting: assume $T_L = T_i$

$$y_L(t) = \frac{2}{\sqrt{\pi}}\frac{T_i - T_0}{\rho_{metal}(-\Delta H)}\sqrt{k_{mould}\rho_{mould}c_p^{mould}}\sqrt{t}$$
$$t_{total} = C\left(\frac{V_{metal}}{A}\right)^2$$ Chvorinov's rule</td>
<td>
</td>
</tr>
<tr>
<td rowspan="2">Poor contact (air gap is present between mould and melt)</td>
<td>General case

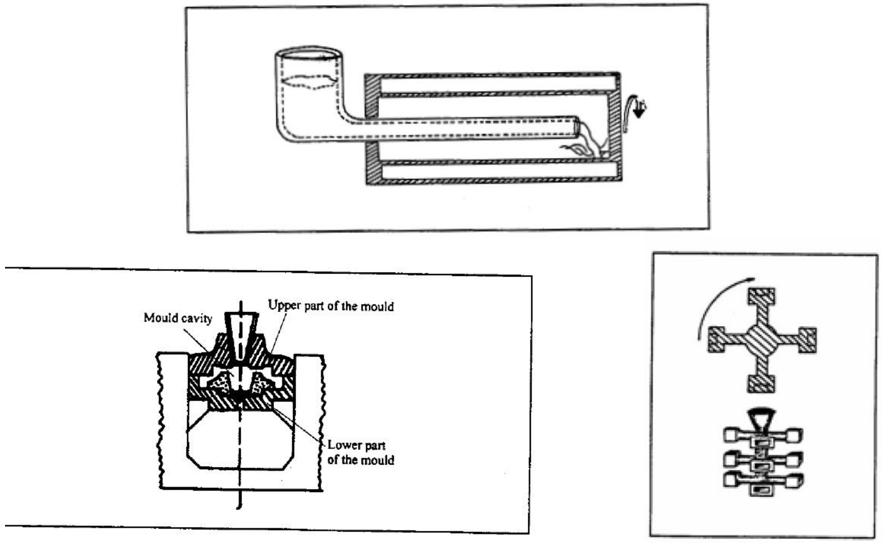
$$t = \frac{\rho(-\Delta H)}{(T_L - T_0)}\left(\frac{y_L}{h}\right)\left(1 + \left(\frac{h}{2k}\right)y_L\right)$$</td>
<td>
</td>
</tr>
<tr>
<td>Special case: Nu<<1 (<0.1)
• Low heat transfer coefficient at mould-solid interface
• Thin solid thickness
• High thermal conductivity of metal

$$t = \frac{\rho(-\Delta H)}{(T_L - T_0)}\left(\frac{y_L}{h}\right)$$</td>
<td>
</td>
</tr>
</table>

Problem 4.2

- Centrifugal casting provides good contact with the mould.
- Cu chill mould has high thermal conductivity.

Centrifugal Casting



https://www.youtube.com/watch?v=zSPaoGwPXlo

4.2 Stainless steel tube castings are often cast in Cu chill-moulds by centrifugal casting. A well-balanced quantity of metal, suitable for casting, is supplied through a channel in the inner part of the chill-mould. The centrifugal force presses the melt towards the chill-mould during the whole casting process. Solidification of the stainless steel melt occurs from the chill-mould surface and inwards towards the centre. The melt is not superheated.

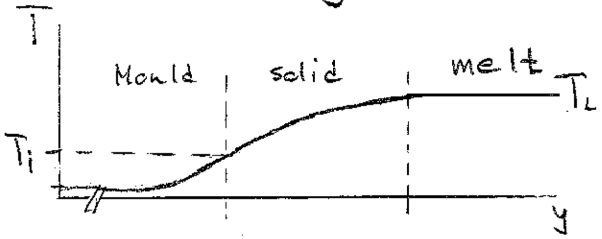
Calculate an approximate value of the solidification time of a tube casting with a thickness of 10 cm. Material constants are found in the table below.

Hint A30

Quantity	Fe (stainless steel)	Cu (chill-mould)
k	30 W/m K (1325 °C)	398 W/m K (25 °C)
ρ_s	7.50×10^3 kg/m^3 (25 °C)	8.94×10^3 kg/m^3 (25 °C)
c_p^s	650 J/kg K (~500 °C)	384 J/kg K (~25 °C)
$-\Delta H$	300 kJ/kg °C	
T	$T_L = 1598$ K (1325 °C) (no excess temperature)	$T_0 = 298$ K (25 °C)

4.2) Approximate the solidification time!

Centrifugal casting in Cu chill mould provides good contact and good thermal conductivity in the mould.



We assume ideal contact:

$y_L(t) = \lambda\sqrt{4\alpha_{metal}\, t}$, α - thermal diffusivity $= \frac{k}{\rho c_p}$

$\lambda = ?$ Can be solved from:

$$\frac{c_p^{metal}(T_L - T_0)}{-\Delta H} = \sqrt{\pi}\,\lambda \exp(\lambda^2)\left(\sqrt{\frac{k_{metal}\rho_{metal}c_p^{metal}}{k_{mould}\rho_{mould}c_p^{mould}}} + \text{erf}\,\lambda\right)$$

Introduce known numbers:

$$2.82 = \left(\sqrt{\pi}\,\lambda \exp(\lambda^2)\right)(0.327 + \text{erf}\,\lambda)$$

Make an educated guess (look at figures 4.14 and 4.15)

$\lambda_1 = 0.80$, get $\text{erf}(\lambda)$ from table

λ_1 gives that the right hand expression

$$\left(\sqrt{\pi}\,\lambda_1 \exp(\lambda_1^2)\right)(0.327 + \text{erf}\,\lambda) = 2.87$$

$2.87 > 2.82$, so try lower value of λ.

Lets try $\lambda_2 = 0.75$

$$\left(\sqrt{\pi}\,\lambda_2 \exp(\lambda_2^2)\right)(0.327 + \text{erf}\,\lambda_2) = 2.42$$

$\lambda_1 > \lambda > \lambda_2$, lets interpolate

$$\frac{\lambda - \lambda_1}{(2.82 - 2.42)} = \frac{\lambda_2 - \lambda_1}{(2.87 - 2.42)} \Rightarrow \lambda = \frac{0.80 - 0.75}{0.45}\cdot 0.4 + 0.75$$

$$\lambda = 0.79$$

So back to first eq

$$y_L(t) = \lambda\sqrt{4\alpha_{metal}\cdot t} = \lambda\sqrt{4\frac{k_{metal}}{\rho_{metal}\cdot c_p^{metal}}}\cdot\sqrt{t}$$

$$t = y_L^2 \cdot \frac{1}{\lambda^2}\cdot\frac{\rho_{metal}\cdot c_p^{metal}}{4\cdot k_{metal}} = \frac{0.1^2}{0.79^2}\cdot\frac{7.5\cdot 10^3\cdot 650}{4\cdot 30}$$

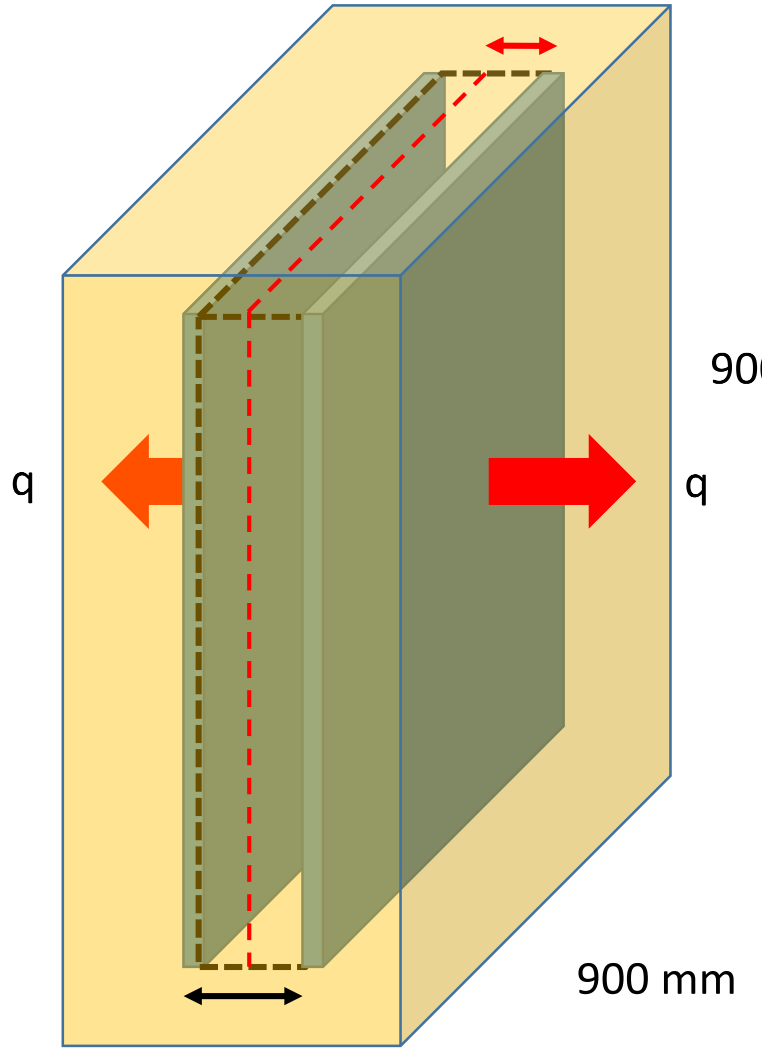
$t = 657\,s \sim \underline{\underline{11\,min}}$

Problem 4.2

λ	λ^2	erf(λ)	$\exp(\lambda^2)$	$\sqrt{\pi}\lambda\exp(\lambda^2)$(0.3272+ erf($\lambda$))
0.8	0.64	1.896	0.7421	2.875
0.79	0.6241	1.866	0.73592	2.77
0.795	0.632	1.8814	0.739	2.826

$$\lambda = 0.795\ (even\ 0.79\ is\ fine)$$

Problem 4.3



Bilateral solidification

The solidification thickness y_L is half of the thickness of the casting.

Chvorinov's rule

$$t = C\left(\frac{V}{A}\right)^2$$

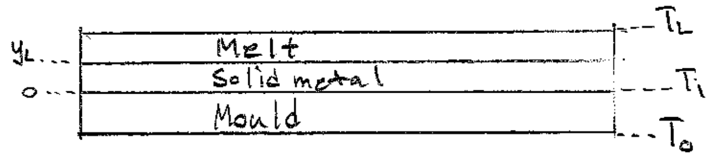
V= Total volume of casting
A= Total available area for heat extraction
C= Material-dependent constant
Take a,b,c as three side length of a rectangular body
V=abc (for a rectangular body)
A= 2(ab+bc+ac)
For thin casting, when c is significantly shorter than a and b, for example
A~ 2ab

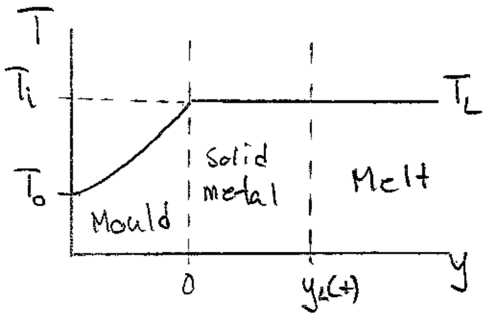
Then

$$t = C\left(\frac{V}{A}\right)^2 = C\left(\frac{abc}{2ab}\right)^2 \sim C\left(\frac{t}{2}\right)^2$$

4.3) Calculate the solidification time!

Pure Al, not super heated, sand mould.





In sand mould there is poor thermal conductivity in the mould relative to the conductivity in the solidified metal. So $T_i = T_L$

Chvorinov's rule!

$$t_{total} = C\left(\frac{V_{metal}}{A}\right)^2$$

where

$$C = \frac{\pi}{4}\frac{\rho_{metal}^2(-\Delta H)^2}{(T_i - T_0)^2 k_{mould}\cdot\rho_{mould}\cdot c_{p,mould}}$$

For Al:

$$C = \frac{\pi}{4}\frac{(2{,}7\cdot10^3\cdot398\cdot10^3)^2}{(933-298)^2}\cdot\frac{1}{0{,}63\cdot1{,}61\cdot10^3\cdot1{,}05\cdot10^3} = 2{,}11\cdot10^6 \; [s/m^2]$$

$$V_{metal} = A\cdot y_L \rightarrow t_{total} = C\left(\frac{A\cdot y_L}{A}\right)^2 = C\cdot y_L^2 \;\left\{\begin{matrix}\text{Bilateral}\\ \text{cooling}\end{matrix}\right\}$$

$$t_{total} = 2{,}11\cdot10^6\cdot0{,}05^2 = 5280\,s \approx \underline{\underline{1{,}47\,h}}$$

For steel:

$$C = \frac{\pi}{4}\frac{(7.88\cdot10^3\cdot272\cdot10^3)^2}{(1808-298)^2}\cdot\frac{1}{0.63\cdot1.61\cdot10^3\cdot1.05\cdot10^3}$$

$$C = 1{,}49\cdot10^6 \; [s/m^2]$$

$$t_{total} = 1{,}49\cdot10^6\cdot0{,}05^2 = 3714\,s \approx \underline{\underline{1{,}03\,h}}$$

(~30 min faster)

Comparison, why is steel faster?

	$(-\Delta H)$	$(T_L - T_0)$	(t_{total})
Aluminium	398	635	1.5h
steel	272	1510	1.0h

Heat of fusion contributes.

But, the heat gradient is much larger for the steel.

Problem 4.6

4.6 In order to increase the production capacity at casting of thin wall Al castings, a foundry has decided to change from sand mould casting to metal mould casting of a product with a thickness of 5.0 mm. The heat transfer coefficient between metal and mould at the mould casting is 900 W/m^2 K. Material data are listed in the table. The room temperature is 20 °C.

Compare the solidification time of the product when cast in a sand mould and in a metal mould.

Hint A40

Hint: Bilateral solidification, poor contact

Stage 1: Very good contact between melt and mould. h is affected by

- Mould wettability by the melt
- Pouring temperature
- Mould roughness
- Mould temperature
- Melt momentum during pouring
- Mould thermal conductivity
- Metallostatic pressure
- Melt turbulence

h value ranged from 2,100 W/m^{2}K for cast iron in a metal mould – 19,000 W/m^{2}K for Al-Si casting on copper chills.

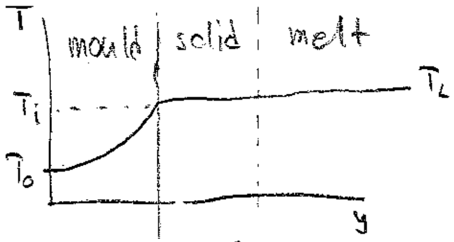
AN Vasileiou, G-C Vosniakos and DI Pantelis. Determination of local heat transfer coefficients in precision castings by genetic optimisation aided by numerical simulation. Proc IMechE Part C: J Mechanical Engineering Science 2015, Vol. 229(4) 735–750.

4.6 Compare solidification time.
Sand vs metal mould.

Solidification time for sand mould we get from Chvorinov's:

$$t_{total} = C\left(\frac{V}{A}\right)^2 , \quad C = \frac{\pi}{4} \frac{\rho_{metal}^2 (-\Delta H)^2}{(T_i - T_0)^2 k_{mould}\, \rho_{mould}\, c_{p\,mould}}$$

$$C = \frac{\pi}{4} \frac{(2.7\cdot10^3)^2 (398\cdot10^3)^2}{(660-20)^2\, 0.63\cdot1.6\cdot10^3\cdot1.05\cdot10^3} = 2.1\cdot10^6 \text{ s/m}^2$$



$$t_{total} = C\cdot\frac{A^2\cdot y_L^2}{A^2} \quad \{y_L = 2.5\cdot10^{-3}\,[m]\}$$

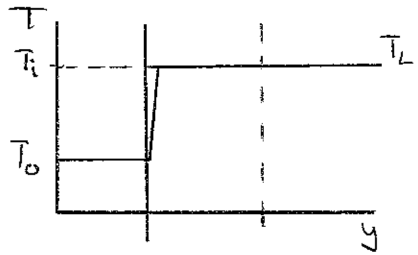
$$t_{total} = 2.1\cdot10^6 \cdot (2.5\cdot10^{-3})^2 = \underline{\underline{13\text{ s}}}$$

Solidification time for metal mould

We have poor contact between solid metal and mould.

$$t = \frac{\rho(-\Delta H)}{T_L - T_0}\cdot\frac{y_L}{h}\left(1 + \frac{h}{2k}\cdot y_L\right)$$

In this case $\frac{hs}{k} \ll 1$, so the temperature distribution will be



We can say this is valid for thin components casted in permanent moulds

$$\text{So}\quad t = \frac{\rho(-\Delta H)}{T_L - T_0}\cdot\frac{y_L}{h}$$

$$t = \frac{2.7\cdot10^3\cdot398\cdot10^3}{660-20}\cdot\frac{2.5\cdot10^{-3}}{900} = \underline{\underline{4.7\text{ s}}}$$

Almost 3 times ~~faster~~ with the metal mould.

Problem 4.9

4.9 It can be seen from Figure 4.1 on page 60 that the heat transport during the solidification process of a casting can be described as a number of steps, coupled in series. The step or steps that correspond to the largest heat transfer resistance will determine the whole temperature distribution.

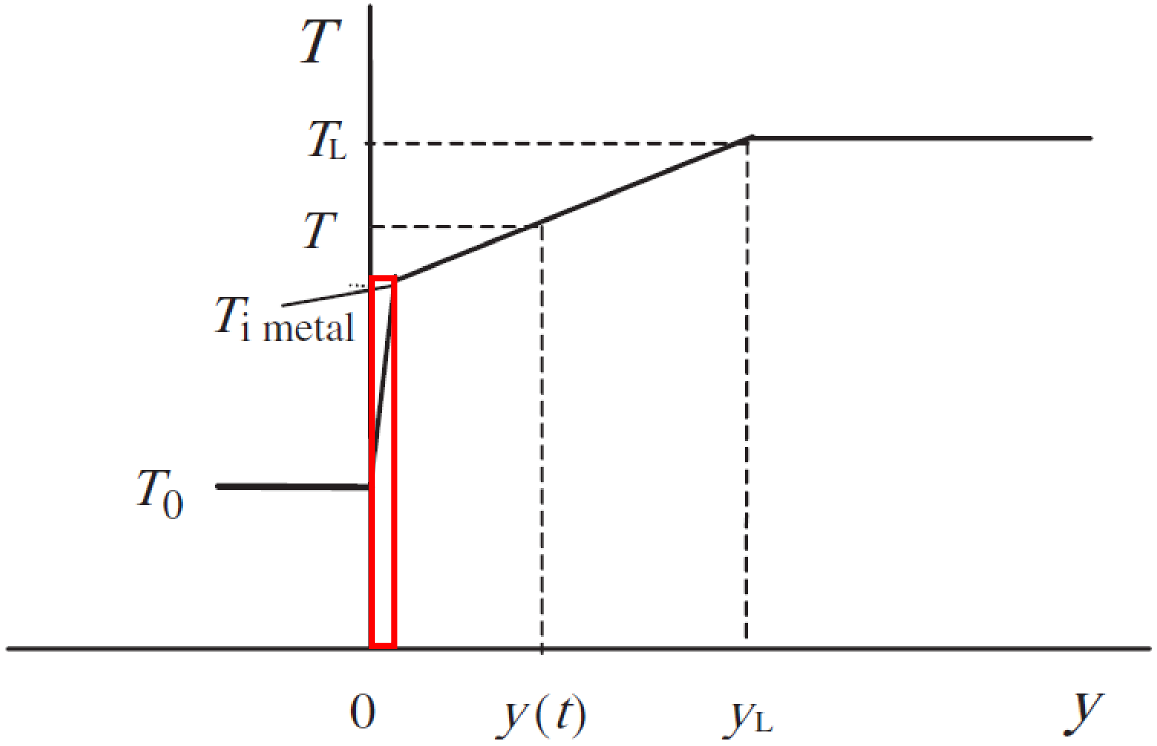
The step that normally offers the largest heat transfer resistance is the air gap between the mould and the casting. Depending on the circumstances, the temperature distribution can either be described by Figure 4.17 on page 73 or by Figure 4.27 on page 86.

The heat transfer coefficient h varies from 2×10^2 up to 2×10^3 W/m^2 K in casting processes of technical interest. The thermal conductivity varies strongly, depending on the choice of alloy.

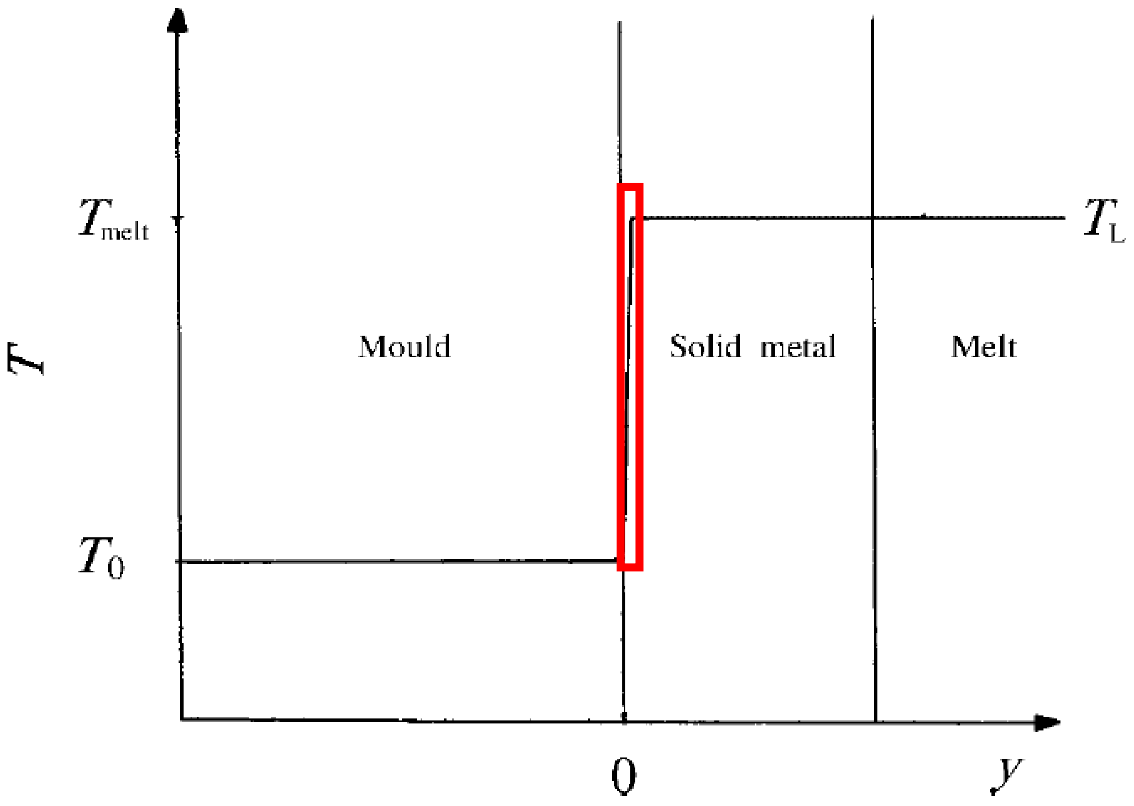
Problem 4.9 (a)

(a) Discuss the conditions for the temperature distributions in Figure 4.17 and Figure 4.27.

Hint A49



Air gap



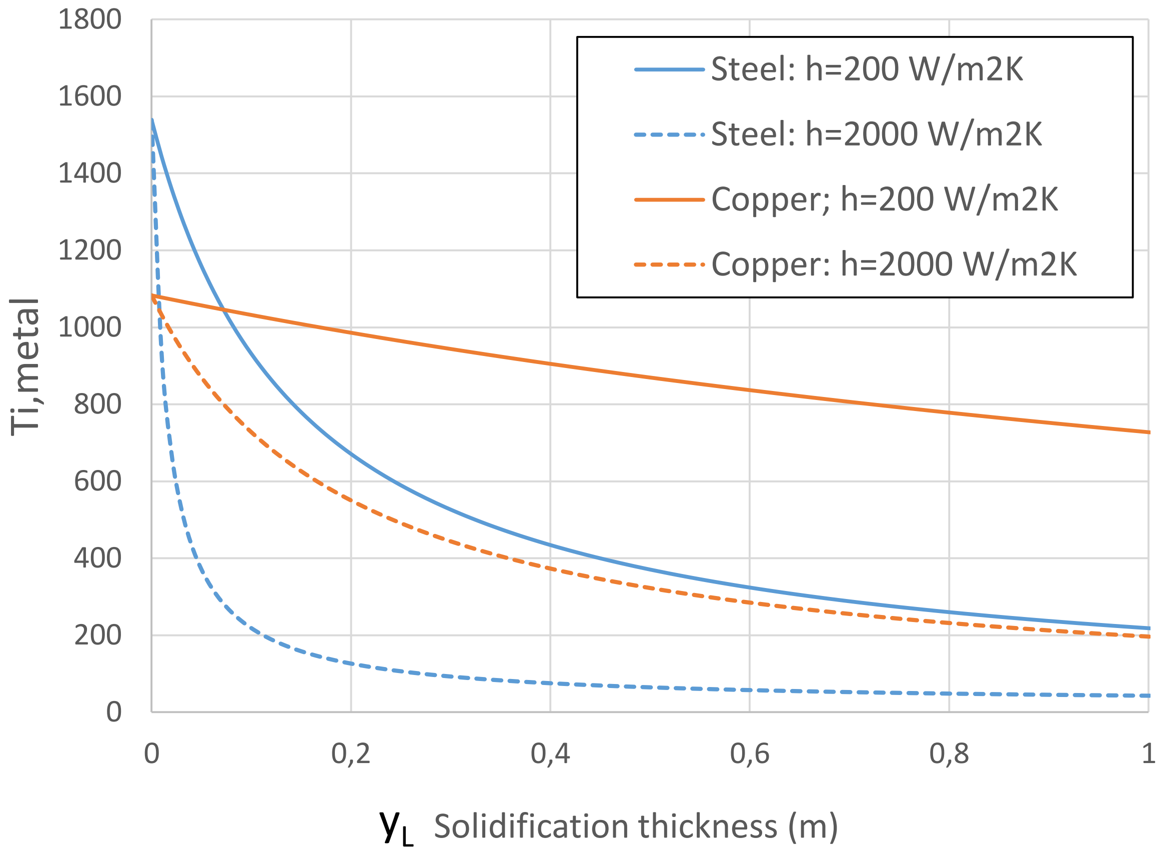
Air gap

Problem 4.9(b)

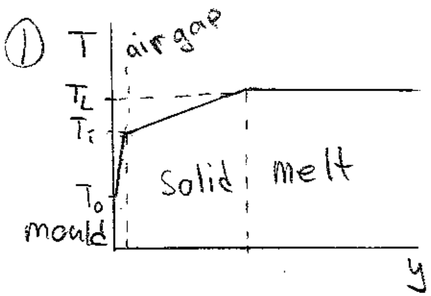
(b) Calculate the surface temperature $T_{i\ metal}$ of steel and copper castings as a function of the thickness y_L of the solidified shell. Use two values of the heat transfer coefficient, $2 \times 10^2\ W/m^2\ K$ and $2 \times 10^3\ W/m^2\ K$ respectively, for the respective metal. Show the results in two diagrams, one for steel and one for copper. The temperature of the surroundings is 20 °C.

Hint A268

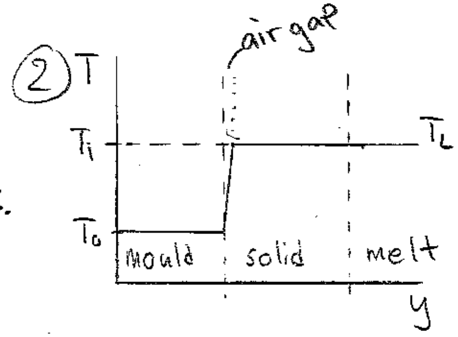
Material constants
Steel:
$T_L = 1530\ °C$
$k\ = 30\ W/m\ K$
Copper:
$T_L = 1083\ °C$
$k\ = 398\ W/m\ K$



4.9) a) Discuss the conditions for the temperature distribution in figure 4.17 and 4.27



vs.



What is different? Which one is valid?

Lets look at eq 4.45, p.74. The formula for poor contact between metal and the mould.

$$T_{i,metal} = \frac{T_L - T_0}{1 + \left(\frac{h}{k}\right) y_L} + T_0$$

Figure ② is a special case.

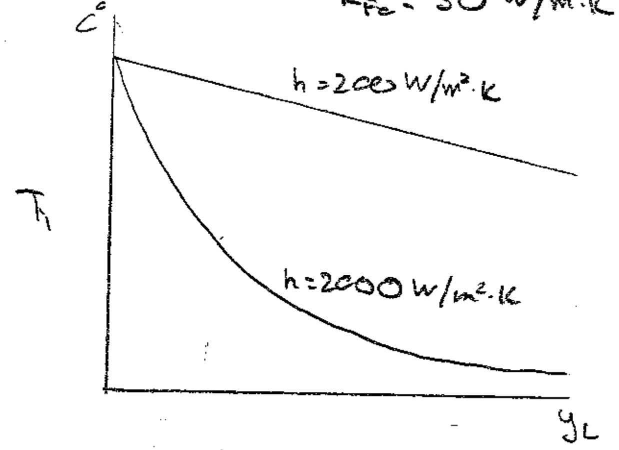
If $\frac{h y_L}{k} \ll 1$ then $T_{i,metal} = \frac{T_L - T_0}{1 + 0} + T_0 = T_L$

$\frac{h s}{k} = Nu$ $\left\{\frac{\text{convective heat transfer}}{\text{conductive heat transfer}}\right\}$

Figure ① is valid for all cases.

For steel: $T_L = 1530°C$

$k_{Fe} = 30\ W/m \cdot K$



For Cu: $T_L = 1083°C$

$k_{Cu} = 398\ W/m \cdot K$