DD2552 - Seminars on Theoretical Computer Science, Programming Languages and Formal Methods, Seminar 4

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Last Seminar and Today

Last seminar:

- more PCTL operators
- deciding PCTL formulas on DTMCs
- intuitions behind CTMCs and CSL

Today:

- CTMCs and CSL in detail
- examples

Continuous Time Markov Chains (CTMCs)

$\mathcal{M} = (S, s_i, Q, L)$ where

- S is a (finite) set of states
- $s_i \in S$ is the initial state
- $Q \colon S \times S \mapsto R^{\geq 0}$ is a matrix where
  - $Q(s_j, s_k)$ is the rate of transition from s_j to s_k
  - $Q(s_j, s_j)$ is constrained to be $-(\sum_{j \neq k} Q(s_k, s_j))$
- $L \colon S \mapsto 2^{\mathrm{AP}}$

Rates of Transition

- let $\mathcal{M} = (S, s_i, Q, L)$
- suppose $Q(s, s') = r$ and $t \in R^{>0}$
- probability of $\mathcal{M}$ moving from s to s' within time t is $1 - e^{-rt}$
- slogan: rate is expentially distributed with parameter $Q(s, s')$
- define $E(s) = \sum_{s' \in S} Q(s, s')$, "total rate"

Example

- $S = s_0, s_1, s_2, s_3$
- $s_i = s_0$
- $L(s_0) = \emptyset$, $L(s_1) = \{\}$, $L(s_2) = \{\}$, $L(s_3) = \{\}$
- \$Q(s_0,s_1) = \$,

Continuous Stochastic Logic (CSL)

$$\phi ::= \top \mid a \mid \neg\phi \mid \phi \wedge \phi \mid P_{\geq\theta}(\psi)$$
$$\psi ::= \phi \, U^{\leq t} \, \phi$$
$$t \in R^{\geq 0}, \quad \theta \in [0, 1]$$

possible additions:

- unbounded until (straightforward)
- next operator (tricky)
- steady state operator (some work)

Paths for CTMCs

- paths π must track the time that was spent in each state
- approach 1: define CTMC path as function from positive reals to states
- approach 2: define CTMC as infinite state sequence with real-valued **transition labels**

$$\pi = s_0 \rightarrow^{t_0} s_1 \rightarrow^{t_1} \ldots$$

with $t_i > 0$ and $Q(s_i, s_{i+1}) > 0$.

Define:

- $\pi(t)$: state at time t)
- $\pi[i]$: the ith state

Semantics of CSL

By "translation" from PCTL:

- $\mathcal{M}, s \models \top$
- $\mathcal{M}, s \models a$
- $\mathcal{M}, s \models \phi \wedge \phi'$
- $\mathcal{M}, s \models \neg\phi$

Tricky ones:

- $\mathcal{M}, \pi \models \phi \; U^{\leq t} \; \phi'$
- $\mathcal{M}, s \models P_{\geq\theta}(\psi)$