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Problems for the seminar 7, MJ2411 HT17

Note: for all the problems, explain where the energy/power balance has been analyzed.

Problem 1. A solar thermal collector will be installed for hot water production in a house in Morocco. You are now hired as a consultant to advise which collector type should be purchased for a design outlet temperature of 65°C and a design total irradiance on the collector of 835W/m^2. Your research on solar collectors for solar thermal applications has yielded the following information:

Collector type **Parameters**	**1.-Uncovered collector**	**2.-Flat plate collector (covered)**
Average transmission glass τ_G^*	-	0.90
Absorption coefficient collector α_A^*	0.95	0.95
U-value front	15 W/(m²K)	5 W/(m²K)
U-value back	2 W/(m²K)	0.5 W/(m²K)
Collector efficiency factor F’	0.935	0.945
Nominal mass flow water	0.25 m³/h	0.25 m³/h
Cost per m²	100 €/m²	170 €/m²

Table 1: Collector specifications

Both collectors can be considered perfectly insulated (adiabatic) at the sides and the ambient temperature equal 20°C. The fluid inlet temperature to the collector can be assumed to be 25°C

For both collectors determine at the design time:

a. The collector area required to achieve the design outlet temperature of 65°C.
b. Which collector would you recommend to buy? Explain why.

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Heat balance of the fluid flowing through the collector:

$$Q_{use} = \dot{M} \cdot c_p \cdot (T_{out} - T_{in})$$

Heat balance on the collector:

$$Q_{use} = (Q_{absob} - Q_{losses}) \cdot F' = I_t \cdot A \cdot F' \cdot \left[(\tau_G^* \alpha_A^*) - U_t \cdot \frac{(T_{fl,m} - T_{amb})}{I_t} \right]$$

$$being \;\; U_t = U_{front} + U_{back} \qquad and \qquad T_{fl,m} = \frac{T_{out} + T_{in}}{2}$$

Combining the formulas and isolating A:

$$A = \frac{\dot{M} \cdot c_p \cdot (T_{out} - T_{in})}{F' \cdot \left[I_t \cdot \tau_G^* \cdot \alpha_A^* - U_t \cdot \left(\frac{T_{out} + T_{in}}{2} - T_{amb} \right) \right]}$$

This formula gives the following results ($\tau_G^* = 1$ for the uncovered collector) :

$$A_{uncovered} = 33.89 \; m^2$$

$$A_{covered} = 21.42 \; m^2$$

The costs can easily be calculated as the product of the specific costs (€/m²) and the collector area

$$C_{uncovered} = 3389 \text{ €}$$

$$C_{covered} = 3641 \text{ €}$$

If cost is the main driver, the uncovered collector would be the most suitable. Nonetheless, if the available space is an issue, the flat plate collector could become a suitable choice.

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Problem 2. The total irradiance on a thermosyphon collector is defined for a certain day with the following formula:

$$I_t = 713.3\cos\left(\frac{(t-12)}{18.15}\cdot\pi\right) + 150\ {}^{W}\!/_{m^2}$$

Where t is the solar time and the cosine is expressed in radians and the length of the day is 18.15h. The collector has a frontal area of 80 m². It is a single glass type with an average transmission τ_G^* of 0.92. The absorption coefficient of the collector α_A^* is 0.98.The collector efficiency factor *F´* can be assumed as 0.96. Other assumptions:

- No thermal losses in the collector and storage tank, and no usage of hot water during the period of operation. The tank water temperature in the morning is the same as outdoor temperature, 15°C. Consider the outdoor temperature constant throughout the day.
- At each instant in time the water in the tank can be considered fully mixed.

What is the tank water capacity if it is known that, if the flat plate collector were perfectly insulated (no heat losses), the tank water temperature at the end of the day would theoretically reach 150 °C (assume there is no phase change and a specific heat capacity of water of 4200J/kgK)?

Problem 3. A solar collector has a frontal area of 80 m². The *U*-value on front is 7 W/(m² K) and on the back 1 W/(m² K). The collector can be considered perfectly insulated (adiabatic) along the sides. The length of the day is 18.15h, the water mass in the tank is 4825kg and a total solar energy absorbed on the collector tubes (not by the water) along the daylight hours is 791kWh. Consider factor *F´* equal to 0.96.

Calculate the heat losses of the collector that day using time-dependent heat losses of the flat plate collector. For the calculation of the losses, consider that the mean fluid temperature is equal to the instantaneous temperature of the fully mixed tank and increases linearly with the time (see Fig. 1). Thus, the mean fluid $T_{fl,m}$ temperature can be assumed to be the initial temperature of the water in the tank at sunrise (ambient temperature) and the fully mixed temperature of the tank at sunset.

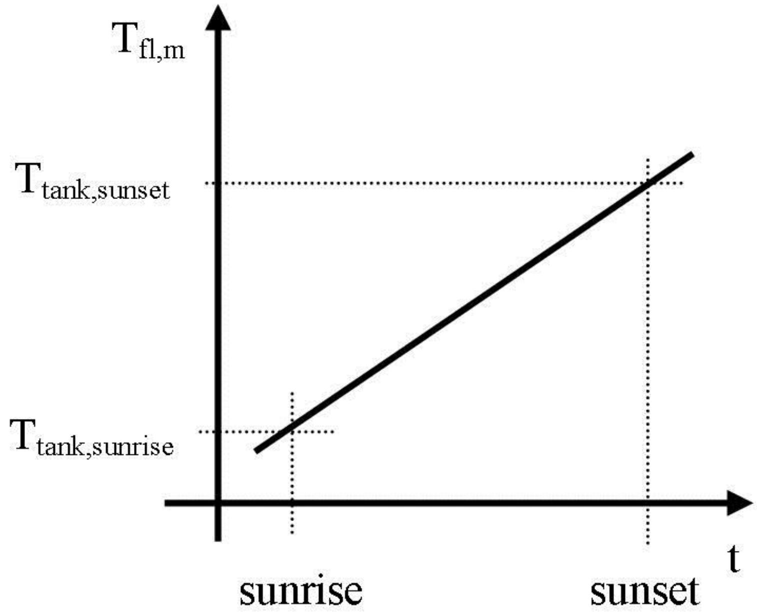

Fig. 1: Mean fluid temperature

Solution problem 2

$$m \cdot c_p \cdot \Delta T = \int_{2.93}^{21.07} \eta \cdot I \cdot A \, dt = F' \cdot \int_{2.93}^{21.07} \dot{Q}_{absorbed} \, dt = F' \cdot \int_{2.93}^{21.07} I \cdot A \cdot \tau \cdot \alpha \cdot dt = F' \cdot Q_{absorbed}$$

$$Q_{absorbed} = \ A \cdot \tau \cdot \alpha \cdot \int_{2.93}^{21.07} \left\{ \varepsilon_{cos} * I_{b,max} \cdot \cos\left(\frac{(t-12)}{N} \cdot \pi \right) + I_{diffuse} \right\} \cdot dt$$

$$Q_{absorbed} = A \cdot \tau \cdot \alpha \cdot \left\{ \int_{2.93}^{21.07} \left[\varepsilon_{cos} * I_{b,max} \cdot \cos\left(\frac{(t-12)}{N} \cdot \pi \right) \right] \cdot dt + \int_{2.93}^{21.07} I_{diffuse} \cdot dt \right\}$$

$$Q_{absorbed} = A \cdot \tau \cdot \alpha \cdot \left\{ \varepsilon_{cos} \cdot I_{b,max} \cdot \left[\sin\left(\frac{t-12}{N} \pi \right) \cdot \frac{N}{\pi} \right]_{t=2.93}^{t=21.07} + \left[I_{diffuse} \cdot t \right]_{t=2.93}^{t=21.07} \right\}$$

$$Q_{absorbed} = A \cdot \tau \cdot \alpha \cdot \left\{ \varepsilon_{cos} \cdot I_{b,max} \cdot 2 \cdot \frac{N}{\pi} + I_{diffuse} \cdot N \right\}$$

$$Q_{absorbed} = 790.8 \, kWh = 2.85 \, GJ$$

$$m = \frac{F' \cdot Q_{absorbed}}{\Delta T \cdot c_p} = \frac{0.96 \cdot 2.85 \cdot 10^9}{(150 - 15) \cdot 4.2 \cdot 10^3} = 4825\ kg$$

Solution problem 3

$$T_{tank,sunset} = T_{end} \text{ (to clarify nomenclature)}$$

$$\dot{Q}_{losses} = U_{total} \cdot A \cdot (T_{fl,m}(t) - T_{amb})$$

Applying Fig.2: $\dot{Q}_{losses} = U_{total} \cdot A \cdot \left[(T_{end} - T_{amb}) \cdot \frac{(t-2.93)}{N}\right]$

Using the reference system of $t_{sunrise}$=0h:

$$\dot{Q}_{losses}' = U_{total} \cdot A \cdot (T_{end} - T_{amb}) \cdot \frac{t}{N}$$

$$Q_{losses} = \int_0^N \dot{Q}_{losses}'\, dt = U_{total} \cdot A \cdot \frac{T_{end} - T_{amb}}{N} \cdot \int_0^N t \cdot dt$$

$$Q_{losses} = \int_0^{18.15} \dot{Q}_{losses}'\, dt = U_{total} \cdot A \cdot \frac{T_{end} - T_{amb}}{2} \cdot N$$

$$T_{end} = T_{start} + \frac{F' \cdot Q_{absorbed}}{m \cdot c_p + \frac{F' \cdot U_{total} \cdot A \cdot N}{2}} = 15 + \frac{0.96 \cdot 2.85 \cdot 10^9}{20.27 \cdot 10^6 + \frac{0.96 \cdot 8 \cdot 80 \cdot 6.53 \cdot 10^4}{2}} = 82.9°C$$

$$Q_{losses} = U_{total} \cdot A \cdot \frac{T_{end} - T_{amb}}{2} \cdot N = 1.42GJ$$

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Problem 4. A solar thermosyphon system has a collector for water heating with a frontal area of 60m². There is a continuous usage of water from the tank keeping its water temperature constant at 35°C. The collector is a single glass type with an average transmission τ_G^* of 0.9. The absorption coefficient of the collector α_A^* is 0.95, the U-value on front is 8 W/(m² K) and on the back 2 W/(m² K). The collector can be considered perfectly insulated (adiabatic) on the sides. The collector efficiency factor F' can be assumed as 0.94. Figure 2 represents the global irradiance approximation received by the collector. The global irradiance in each section is constant and equal to 265, 677 and 251 W/m^2 for section I, II and III, respectively. Important assumption:

- There is no thermal loss in the water tank. The outdoor temperature is 20°C.
- Consider a mass flow through the solar collector of 0.04 kg/s during the whole day.

a. How much heat (thermal energy) is collected by the thermosyphon during the whole day?

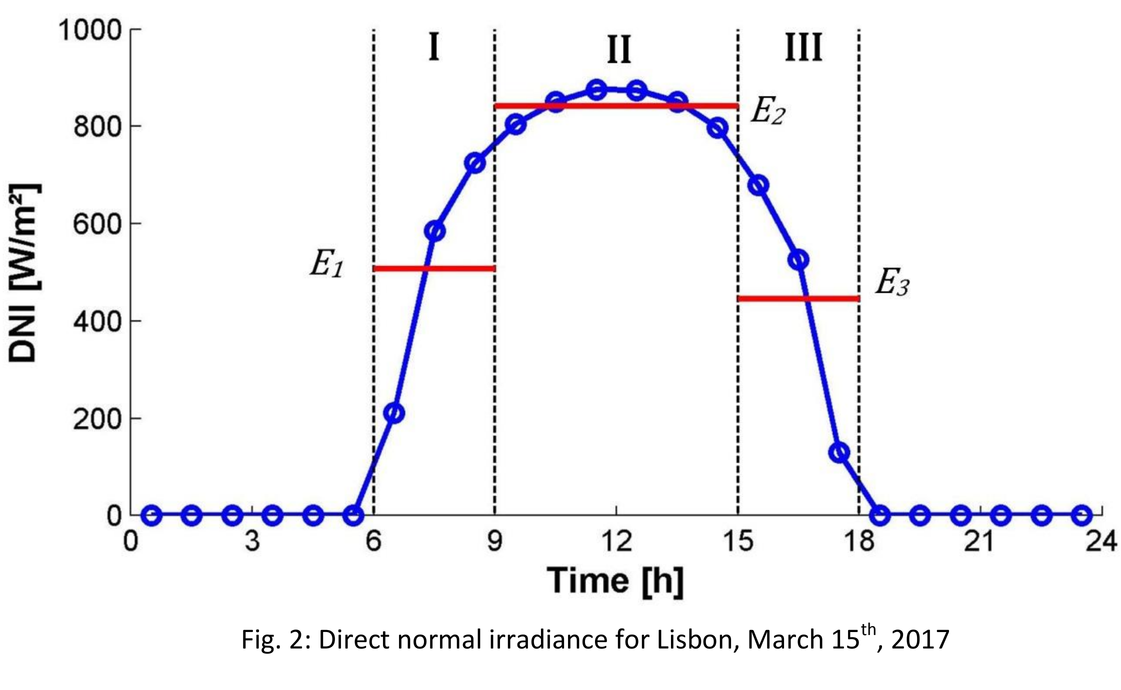

Fig. 2: Direct normal irradiance for Lisbon, March 15th, 2017

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Solution problem 4:

Heat balance of the fluid flowing through the collector:

$$\dot{Q}_{use} = \dot{M} \cdot c_p \cdot (T_{out} - T_{in})$$

Heat balance on the collector:

$$\dot{Q}_{use} = (\dot{Q}_{absob} - \dot{Q}_{losses}) \cdot F' = A \cdot F' \cdot \left[I_t \cdot \tau_G^* \cdot \alpha_A^* - U_t \cdot (T_{fl,m} - T_{amb})\right]$$

$$being\ \ U_t = U_{front} + U_{back} \qquad and \qquad T_{fl,m} = \frac{T_{out} + T_{in}}{2}$$

Combining the formulas and isolating T_{out}:

$$T_{out} = \frac{A \cdot F' \cdot I_t \cdot \tau_G^* \cdot \alpha_A^* + \dot{M} \cdot c_p \cdot T_{in} + U_t \cdot A \cdot F' \cdot \left(T_{amb} - \frac{T_{in}}{2}\right)}{\dot{M} \cdot c_p + U_t \cdot A \cdot F' \cdot \frac{1}{2}}$$

This formula gives the following results of T_{out} for the 3 regions:

$$T_{out,1} = 44.64°C$$

$$T_{out,2} = 88.73°C$$

$$T_{out,3} = 43.11°C$$

$$\dot{Q}_{use,1} = \dot{M} \cdot c_p \cdot \left(T_{out,1} - T_{in}\right) = 1.62\ kW$$

$$\dot{Q}_{use,2} = \dot{M} \cdot c_p \cdot \left(T_{out,2} - T_{in}\right) = 9.03\ kW$$

$$\dot{Q}_{use,3} = \dot{M} \cdot c_p \cdot \left(T_{out,1} - T_{in}\right) = 1.36\ kW$$

Then, the total heat collected can be calculated as:

$$Q_{use} = \dot{Q}_{use,1} \cdot \Delta t_1 + \dot{Q}_{use,2} \cdot \Delta t_2 + \dot{Q}_{use,3} \cdot \Delta t_3 = 63.1\ kWh$$