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Problems for the seminar "solar fundamentals", MJ2411 HT17

Problem 1. The solar angles affecting a thermosyphon system for hot water production are to be analyzed on April 10th 2016 (summer time) in Malmö. The solar collector of the thermosyphon system is placed due south with a tilt angle equal to 55°. The solar angles are assessed at 11:00 (clock time).

What is the cosine effectiveness of the solar collector at 11.00?

Problem 2. A characteristic DNI (beam radiation) measurement along a day is considered as shown in Fig. 1 (dashed line). In order to simplify the calculations, the beam radiation (I_b) can be approximated during the daylight hours via the equation below, where t is the solar time and N the length of the day in hours. The approximated beam radiation is shown by the solid line.

$$I_b = I_{b,max} \cdot \cos\left(\frac{(t-12)}{N} \cdot \pi\right) \text{ W/m}^2 \text{ (cosine defined in radians)}$$

The total horizontal irradiance and the diffuse irradiance were at solar noon 650 and 100W/m², respectively. What is the maximum beam irradiance $I_{b,max}$? Determine the formula of the total irradiance on the collector during the daylight hours as a function of time.

Assumptions:

- Diffusive radiation is constant throughout the day.
- Consider the cosine effectiveness on the solar collector constant during the day and equal to 0.90.
- Assume a reasonable value if there is any missing data.

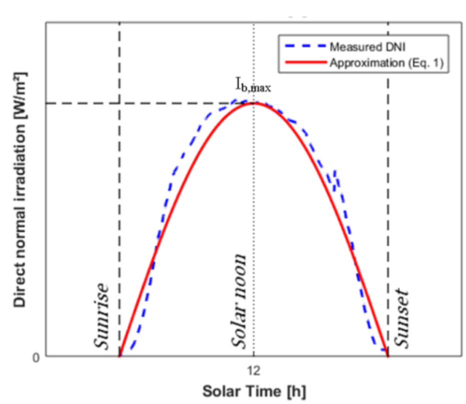

Fig. 1: Direct normal irradiance in Stockholm

KTH VETENSKAP OCH KONST

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Problem 3. April 21st 2015 at solar noon is considered for the performance calculations of the collector in Stockholm, with the solar collector system placed due south at an inclination angle $\beta_c = 56°$.

a. Explain in a few words the value of the angles implicitly defined in the problem description.
b. How many hours of sunlight are registered on April 21st in Stockholm? What solar time does the sun rise and set?
c. What is the cosine effectiveness of the solar collector at the design time?

Problem 4. What should the position of a solar collector be (azimuth and tilt angles) to maximize the cosine effectiveness at each time (provide general explanation, no calculation)?

A shadow-band pyranometer indicates irradiances of $70 W/m^2$ and $450 W/m^2$. If the incidence surface angle on a surface is 33° and the zenith angle 54.5°, what is the total irradiance on the collector?

Solution

1. Malmö is located 55°36'20''N 13°E (UTC+1)

$n = 31 + 29 + 31 + 10 = 101$ (The solution was also considered correct for n=100)

$\Psi_{\mathrm{loc}} = -13°W$ (since it is to the east)

For Mälmo the time zone meridian can be calculated as -UTC*15° :

$$\Psi_{\mathrm{std}} = -15°W$$

The equation of time on April 10th:

$$\Delta t_{EOT} = A \cdot \cos\left(2\pi \frac{n-1}{365}\right) + B \cdot \sin\left(2\pi \frac{n-1}{365}\right) + C \cdot \cos\left(4\pi \frac{n-1}{365}\right) + D \cdot \sin\left(4\pi \frac{n-1}{365}\right)$$

$$\Delta t_{EOT} = -1.15\ [min], \quad for\ n = 101$$

The solar time at solar noon can be calculated as

$$t_s = t_{clock} + \frac{(\Psi_{\mathrm{std}}) - (\Psi_{\mathrm{loc}})}{15°} + \frac{(\Delta t_{EOT}[min])}{60} + (\Delta t_{DST}) =$$

$t_s = 11 + \frac{(-15°)-(-(-13°))}{15°} + \frac{(-1.15)}{60} + (-1) = 9.85\,h$= 9h 51' 04'' (Note that this was incorrect in the seminar presentation; the daylight saving time is negative)

With the declination angle and latitude being

$$\delta = \arcsin\left(0.39795 \cdot \cos\left(2\pi \frac{n-173}{365}\right)\right) = 7.44° = 0.13\,rad\ for\ n = 101$$

$$\varphi = 55.61° = 0.97\,rad$$

$$\beta_c = 55° = 0.96\,rad$$

To obtain cosine effectiveness:

$$\omega = \frac{\pi}{12}(t_s - 12) = -32.29° = -0.564\,rad$$

$$\theta_z = \arccos(cos\,\delta\,cos\,\varphi\,cos\,\omega + sin\delta\,sin\varphi) = 54.52° = 0.952\,rad$$

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$$\gamma_s = \mathrm{sgn}(\omega)\left|\arccos\left(\frac{\cos\theta_z \sin\varphi - \sin\delta}{\sin\theta_z \cos\varphi}\right)\right| = -40.6° = -0.71 \text{ rad}$$

$$\gamma_c = 0° \; with\; the\; collector\; due\; south$$

$$\theta = \arccos(cos\,\beta_c\, cos\,\theta_z + sin\beta_c sin\theta_z\, cos(\gamma_s - \gamma_c)) = 32.9° = 0.574\, rad$$

$$\varepsilon_{cos} = \cos(\theta) = 0.84$$

2.

Reasonable values assumed (discussed in the seminar):

$$\theta_z = 48.2°$$

$$N = 14.5\, h$$

$$I_{t,h} = I_b \cos\theta_z + I_{d,h} \quad \text{(at solar noon)}\; I_b = I_{b,\max} = \frac{I_{t\max,h} - I_{d,h}}{\cos\theta_z} = \frac{650-100}{0.666} = 826\,\text{W/m}^2$$

$$I_t = \varepsilon_{cos} * I_{b,max} \cdot \cos\left(\frac{(t-12)}{N} \cdot \pi\right) + I_{diffuse} = 744 \cdot \cos\left(\frac{(t-12)}{14.5} \cdot \pi\right) + 100\ \text{W/m}^2$$

3.

n=31+28+31+21=111

$$\varphi = 59.33°$$

With the declination angle being

$$\delta = \arcsin\left(0.39795 \cdot \cos\left(2\pi\frac{n-173}{365}\right)\right) = 11.07° = 0.193 rad\; for\; n = 111$$

The length of the day is:

$$N = \frac{24}{\pi}\arccos(-\,tan(\varphi)\,tan(\delta)) = 14.57\, h$$

The sun time at sunrise and sunset are:

$$t_{solar,sunrise} = 12 - \frac{N}{2} = 4.72h$$

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$t_{solar,sunset} = 12 + \frac{N}{2} = 19.28h$

$$\beta_c = 56° = 0.9774\, rad$$

$$\gamma_s = 0°\ and\ \omega = 0\ \ at\ solar\ noon$$

$$\gamma_c = 0°\ with\ the\ collector\ due\ south$$

$$\theta_z = \arccos(cos\,\delta\, cos\,\varphi\, cos\,\omega + sin\delta\, sin\varphi) = \varphi - \delta = 48.26° = 0.84 rad\ at\ solar\ noon$$

$$\theta = \arccos(cos\,\beta_c\, cos\,\theta_z + sin\beta_c sin\theta_z\, cos(\gamma_s - \gamma_c)) = \beta_c - \theta_z = 7.74° = 0.135 rad\ at\ solar\ noon$$

$$\varepsilon_{cos} = \cos(\theta) = 0.991$$

4. To maximize the cosine effectiveness at any time, the collector normal vector must be facing the sun (like with tracking technologies). Then, the tilt angle should be equal to the zenith angle and the collector azimuth angle equal to the solar azimuth angle. It can also be obtained mathematically with the formula of the angle of incidence.

$$\beta_{c,2} = \theta_z$$

$$\gamma_{c,2} = \gamma_s$$

A shadow-band pyranometer measures alternatingly the diffuse irradiance and the global horizontal irradiance so:

$$I_{d,h} = 70 W/\mathrm{m}^2$$

$$I_{t,h} = 450 W/\mathrm{m}^2$$

$$I_{t,h} = I_b \cos\theta_z + I_{d,h} \Rightarrow I_b = \frac{I_{t,h} - I_{d,h}}{\cos\theta_z} = \frac{450 - 70}{0.58} = 655\ \mathrm{W/m^2}$$

$$I_{t,c} = \varepsilon_{cos} * I_b + I_{diffuse} = 620\ \mathrm{W/m^2}$$