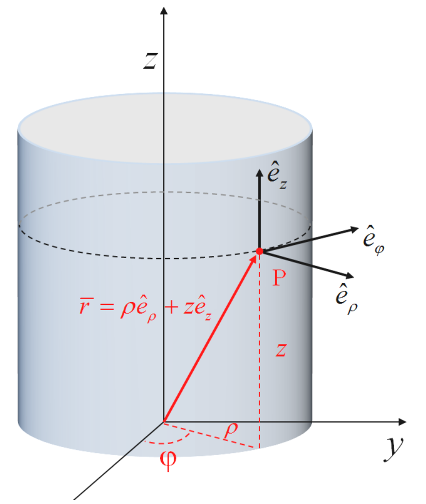
CURVILINEAR COORDINATES (u_1, u_2, u_3)

Base vectors	$\hat{e}_i = \frac{1}{h_i}\frac{\partial \bar{r}}{\partial u_i}$ *with scale factor* $h_i = \left\lvert \frac{\partial \bar{r}}{\partial u_i} \right\rvert$
Position vector differential	$d\bar{r} = \sum_{i=1}^{3} h_i du_i \hat{e}_i$
Surface element	$dS_3 = h_1 h_2 du_1 du_2 \quad (dS_3 \text{ surface perpendicular to } u_3 \text{ axis})$
Volume element	$dV = h_1 h_2 h_3 du_1 du_2 du_3$
Gradient	$\nabla\phi = \sum_i \frac{1}{h_i}\frac{\partial \phi}{\partial u_i}\hat{e}_i$
Divergence	$\nabla\cdot\bar{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(A_1 h_2 h_3) + \frac{\partial}{\partial u_2}(A_2 h_3 h_1) + \frac{\partial}{\partial u_3}(A_3 h_1 h_2)\right]$
Curl	$\nabla\times\bar{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$

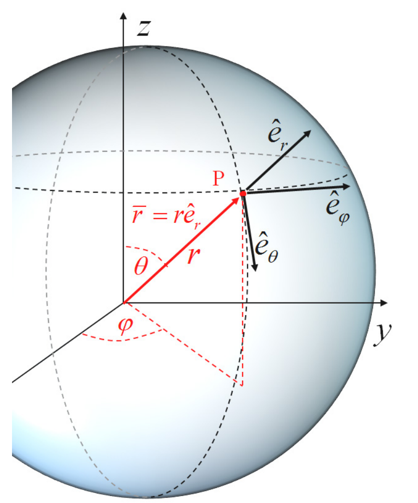
CYLINDRICAL COORDINATE SYSTEM



CYLINDRICAL COORDINATES

Position vector	$\bar{r} = (\rho\cos\varphi, \rho\sin\varphi, z) = \rho\cos\varphi\,\hat{e}_x + \rho\sin\varphi\,\hat{e}_y + z\,\hat{e}_z$ $\bar{r} = \rho\,\hat{e}_\rho + z\,\hat{e}_z$
Scale factors	$h_\rho = 1, \qquad h_\varphi = \rho, \qquad h_z = 1$
Base vectors	$\hat{e}_\rho = (\cos\varphi, \sin\varphi, 0) = \cos\varphi\,\hat{e}_x + \sin\varphi\,\hat{e}_y$ $\hat{e}_\varphi = (-\sin\varphi, \cos\varphi, 0) = -\sin\varphi\,\hat{e}_x + \cos\varphi\,\hat{e}_y$ $\hat{e}_z = (0,0,1)$
Surface element	$dS_\rho = \rho d\varphi dz$ $dS_z = \rho d\varphi d\rho$
Volume element	$dV = \rho d\varphi d\rho dz$
Gradient	$\nabla\phi = \left(\frac{\partial\phi}{\partial\rho}, \frac{1}{\rho}\frac{\partial\phi}{\partial\varphi}, \frac{\partial\phi}{\partial z}\right) = \frac{\partial\phi}{\partial\rho}\hat{e}_\rho + \frac{1}{\rho}\frac{\partial\phi}{\partial\varphi}\hat{e}_\varphi + \frac{\partial\phi}{\partial z}\hat{e}_z$
Divergence	$\nabla\cdot\bar{A} = \left(\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}\right)$
Curl	$\nabla\times\bar{A} = \left(\frac{\partial A_z}{\rho\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{e}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\hat{e}_\varphi + \left(\frac{\partial(\rho A_\varphi)}{\rho\partial\rho} - \frac{\partial A_\rho}{\rho\partial\varphi}\right)\hat{e}_z$
Laplacian	$\nabla^2\phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$

SPHERICAL COORDINATE SYSTEM



SPHERICAL COORDINATES

Position vector	$\bar{r} = (r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta) = r\sin\theta\cos\varphi\,\hat{e}_x + r\sin\theta\sin\varphi\,\hat{e}_y + r\cos\theta\,\hat{e}_z$ $\bar{r} = r\,\hat{e}_r$
Scale factors	$h_r = 1, \qquad h_\theta = r, \qquad h_\varphi = r\sin\theta$
Base vectors	$\hat{e}_r = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) = (\sin\theta\cos\varphi)\hat{e}_x + (\sin\theta\sin\varphi)\hat{e}_y + \cos\theta\,\hat{e}_z$ $\hat{e}_\theta = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta) = (\cos\theta\cos\varphi)\hat{e}_x + (\cos\theta\sin\varphi)\hat{e}_y - \sin\theta\,\hat{e}_z$ $\hat{e}_\varphi = (-\sin\varphi, \cos\varphi, 0) = -\sin\varphi\,\hat{e}_x + \cos\varphi\,\hat{e}_y$
Surface element	$dS_r = r^2\sin\theta d\theta d\varphi$
Volume element	$dV = r^2\sin\theta d\theta d\varphi dr$
Gradient	$\nabla\phi = \left(\frac{\partial\phi}{\partial r}, \frac{1}{r}\frac{\partial\phi}{\partial\theta}, \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}\right) = \frac{\partial\phi}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}\hat{e}_\varphi$
Divergence	$\nabla\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(A_\varphi)$
Curl	$\nabla\times\bar{A} = \left(\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\varphi) - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial\varphi}\right)\hat{e}_r + \left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)\right)\hat{e}_\theta + \left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial\theta}\right)\hat{e}_\varphi$
Laplacian	$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}$

VEKTORALGEBRA FORMLER, NABLA OCH INDEXRÄKNING

$\bar{a} = (a_x, a_y, a_z)$ i kartesiska koordinater

$\bar{b} = (b_x, b_y, b_z)$ i kartesiska koordinater

$\hat{n} = (n_x, n_y, n_z)$ i kartesiska koordinater

$|\hat{n}| = 1$

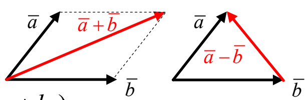
$\bar{a} = \bar{b}$ innebär att $a_x = b_x$ cykl., dvs vektorerna $\bar{a}$ och $\bar{b}$ sammanfaller efter parallell förflyttning.

$|\bar{a}| = \sqrt{\bar{a} \cdot \bar{a}} = \sqrt{a_x^2 + a_y^2 + a_z^2}$ är beloppet (dvs längden) av vektorn $\bar{a}$.

Kartesiska enhetsvektorer:

$\hat{e}_x = (1,0,0)$ $\hat{e}_y = (0,1,0)$ $\hat{e}_z = (0,0,1)$

$\bar{a} = a_x\hat{e}_x + a_y\hat{e}_y + a_z\hat{e}_z$



$\bar{a} + \bar{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$

$\bar{a} - \bar{b} = \bar{a} + \left(-\bar{b}\right) = (a_x - b_x, a_y - b_y, a_z - b_z)$

$c\bar{a} = (ca_x, ca_y, ca_z)$

$\bar{a} + \bar{b} = \bar{b} + \bar{a}$

$\left(\bar{a} + \bar{b}\right) + \bar{c} = \bar{a} + \left(\bar{b} + \bar{c}\right)$

$c\left(\bar{a} + \bar{b}\right) = c\bar{a} + c\bar{b}$

$(s+t)\bar{a} = s\bar{a} + t\bar{a}$

Skalärprodukten:

$\bar{a} \cdot \bar{b} = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$

$\bar{a} \cdot \bar{b} = |a||b|\cos\alpha \quad \Rightarrow \quad \cos\alpha = \dfrac{\bar{a} \cdot \bar{b}}{|a||b|}$

$\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$

$\left(s\bar{a}\right) \cdot \left(t\bar{b}\right) = st\left(\bar{a} \cdot \bar{b}\right)$

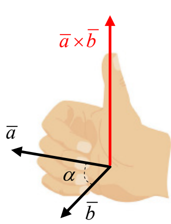
$\bar{a} \cdot \left(\bar{b} + \bar{c}\right) = \bar{a} \cdot \bar{b} + \bar{a} \cdot \bar{c}$

$\bar{a} \cdot \bar{a} = |\bar{a}|^2$

$\bar{a} \cdot \bar{b} = 0 \Leftrightarrow \bar{a} \perp \bar{b}$

Kryssprodukten

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



$\left|\bar{a} \times \bar{b}\right| = |\bar{a}||\bar{b}|\sin\alpha$

Vektorn $\bar{a} \times \bar{b}$ är vinkelrät mot $\bar{a}$ och $\bar{b}$

$\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$

$\left(s\bar{a}\right) \times \left(t\bar{b}\right) = st\left(\bar{a} \times \bar{b}\right)$

$\bar{a} \times \left(\bar{b} + \bar{c}\right) = \bar{a} \times \bar{b} + \bar{a} \times \bar{c}$

$\bar{a} \times \bar{a} = 0$

$\hat{e}_x \times \hat{e}_y = \hat{e}_z \quad \hat{e}_y \times \hat{e}_z = \hat{e}_x \quad \hat{e}_z \times \hat{e}_x = \hat{e}_y$

Trippelprodukter: $\bar{a} \cdot \left(\bar{b} \times \bar{c}\right) = \bar{b} \cdot \left(\bar{c} \times \bar{a}\right) = \bar{c} \cdot \left(\bar{a} \times \bar{b}\right)$

$$\bar{a} \cdot \left(\bar{b} \times \bar{c}\right) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

bac-cab regeln: $\bar{a} \times \left(\bar{b} \times \bar{c}\right) = \bar{b}\left(\bar{a} \cdot \bar{c}\right) - \bar{c}\left(\bar{a} \cdot \bar{b}\right)$

$\left(\bar{a} \times \bar{b}\right) \times \bar{c} = \left(\bar{a} \cdot \bar{c}\right)\bar{b} - \left(\bar{b} \cdot \bar{c}\right)\bar{a}$

Derivator i vektoranalysen:

Här är $\bar{A}$ och $\bar{B}$ vektorfält, medan Φ och Ψ är skalärfält, dvs funktioner av läget i rummet $\bar{r} = (x, y, z)$.

$\nabla = \left(\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z}\right) = \dfrac{\partial}{\partial x}\hat{e}_x + \dfrac{\partial}{\partial y}\hat{e}_y + \dfrac{\partial}{\partial z}\hat{e}_z$ *i kartesiska koordinater*

Linjäritet :

$\nabla\left(t\Phi + s\Psi\right) = t\nabla\Phi + s\nabla\Psi$

$\nabla \cdot \left(t\bar{A} + s\bar{B}\right) = t\left(\nabla \cdot \bar{A}\right) + s\left(\nabla \cdot \bar{B}\right)$

$\nabla \times \left(t\bar{A} + s\bar{B}\right) = t\left(\nabla \times \bar{A}\right) + s\left(\nabla \times \bar{B}\right)$

Produktregler:

$\nabla\left(\Phi\Psi\right) = \Phi\nabla\Psi + \Psi\nabla\Phi$

$\nabla\left(\bar{A} \cdot \bar{B}\right) = \bar{A} \times \left(\nabla \times \bar{B}\right) + \bar{B} \times \left(\nabla \times \bar{A}\right) + \left(\bar{A} \cdot \nabla\right)\bar{B} + \left(\bar{B} \cdot \nabla\right)\bar{A}$

$\nabla \cdot \left(\Phi\bar{A}\right) = \Phi\left(\nabla \cdot \bar{A}\right) + \bar{A} \cdot \left(\nabla\Phi\right)$

$\nabla \cdot \left(\bar{A} \times \bar{B}\right) = \bar{B} \cdot \left(\nabla \times \bar{A}\right) - \bar{A} \cdot \left(\nabla \times \bar{B}\right)$

$\nabla \times \left(\Phi\bar{A}\right) = \Phi\left(\nabla \times \bar{A}\right) - \bar{A} \times \left(\nabla\Phi\right)$

$\nabla \times \left(\bar{A} \times \bar{B}\right) = \left(\bar{B} \cdot \nabla\right)\bar{A} - \left(\bar{A} \cdot \nabla\right)\bar{B} + \bar{A}\left(\nabla \cdot \bar{B}\right) - \bar{B}\left(\nabla \cdot \bar{A}\right)$

Andraderivator

$\nabla \cdot \left(\nabla \times \bar{A}\right) = 0$

$\nabla \times \left(\nabla\Phi\right) = 0$

$\nabla \times \left(\nabla \times \bar{A}\right) = \nabla\left(\nabla \cdot \bar{A}\right) - \nabla^2\bar{A}$

Indexräkning

$\bar{a} \cdot \bar{b} = a_i b_i$

$\left(\bar{a} \times \bar{b}\right)_i = \varepsilon_{ijk} a_j b_k$

$\varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{kij}$ och $\varepsilon_{ijk} = -\varepsilon_{jik}$

$\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & otherwise \end{cases}$ och $\begin{cases} \delta_{ii} = 3 \\ \delta_{km} l_{jm} = l_{jk} \end{cases}$

$\left(\nabla\phi\right)_i = \phi_{,i} \quad \nabla \cdot \bar{A} = A_{i,i} \quad \left(\nabla \times \bar{A}\right)_i = \varepsilon_{ijk} A_{k,j}$

Point source: $\begin{cases} \bar{A}(\bar{r}) = \dfrac{s}{r^2}\hat{e}_r \text{ eller } \bar{A}(\bar{r}) = s\dfrac{\bar{r} - \bar{r}_0}{\left|\bar{r} - \bar{r}_0\right|^3} \text{ med källen på } \bar{r}_0 \\ \bar{A} = grad\phi \quad \Rightarrow \quad \phi = -\dfrac{s}{r} + c \\ \displaystyle\oiint_S \dfrac{s}{r^2}\hat{e}_r \cdot d\bar{S} = \begin{cases} 0 & \text{om källen yttre punkt i V} \\ 4\pi s & \text{om källen inre punkt i V} \end{cases} \end{cases}$

Dipole source: $\phi = \dfrac{\bar{p} \cdot \bar{r}}{r^3}$ och $\bar{E}\left(\bar{r}\right) = -\dfrac{\bar{p}}{r^3} + \dfrac{3\left(\bar{p} \cdot \bar{r}\right)\bar{r}}{r^5}$

Vortex: $\displaystyle\oint_L \dfrac{k}{\rho}\hat{e}_\varphi \cdot d\bar{r} = 2\pi kN$