GAUSS's THEOREM.
Logic steps to prove the theorem

1- Consider a closed surface.

2- Divide the surface in two parts, an upper surface and a lower surface and consider an infinitesimal surface element dS_1 on the upper surface.

3- Consider the projection of the surface element on the xy plane, it will be dxdy. The projection will identify a infinitesimal surface element (dS_2) on the lower surface.

4- Write the expression that relates dxdy to dS_1 and dS_2.

5- Write down the volume integral of $div\bar{A}$

6- Split the volume integral into three terms.

  6.1- Consider only the term which depends on the z-derivative of A_z.

  6.2- Remove the z-derivative by solving the integral in dz.
  What will remain is just the integral in dxdy.

  6.3- Express dxdy in order to obtain dS_1 and dS_2.

  6.4- Re-arrange the integrals in dS_1 and dS_2 in order to have obtain
  a flux integral of $(0,0,A_z)$.

7- Repeat the same for the terms which depend on the x-derivative of A_x and on the y-derivative of A_y.

8- Add all the three terms together in order to obtain the flux of $\bar{A}$.

STOKES' THEOREM.
Logic steps to prove the theorem

1- Consider a closed path and a surface whose boundary is defined by the closed path.

2- Divide the surface in small areas S^i and consider the projection of S^i on the xy, yz, xz planes

3- Prove the Stokes' theorem on S^i_z:

3.1 Write the line integral of the vector field along the boundary of S^i_z and split the integral into three terms.

3.2 Consider only the integral in dx and prove that $\int_{L^i_z} A_x(x,y,z_0)dx = -\iint_{S^i_z} \frac{\partial A_x}{\partial y} dxdy$

3.3 Repeat the same for the integral in dy and dz

3.4 Add the three integrals in dx, dy and dz to obtain $\int_{L^i_z} \overline{A} \cdot d\overline{r} = \iint_{S^i_z} (rot\overline{A})_z dxdy$

3.5 Rewrite $dxdy$ to obtain $\int_{L^i_z} \overline{A} \cdot d\overline{r} = \iint_{S^i} (rot\overline{A})_z \hat{e}_z \cdot d\overline{S}$

4- Prove the Stokes' theorem on S^i:

4.1 Repeat the same procedure for S^i_x and S^i_y

4.1 add together the expressions for the integrals in S^i_x to S^i_y and S^i_z obtaining: $\int_{L^i} \overline{A} \cdot d\overline{r} = \iint_{S^i} rot\overline{A} \cdot d\overline{S}$

5- Prove the Stokes' theorem on S: add together all the expressions obtained for S^i