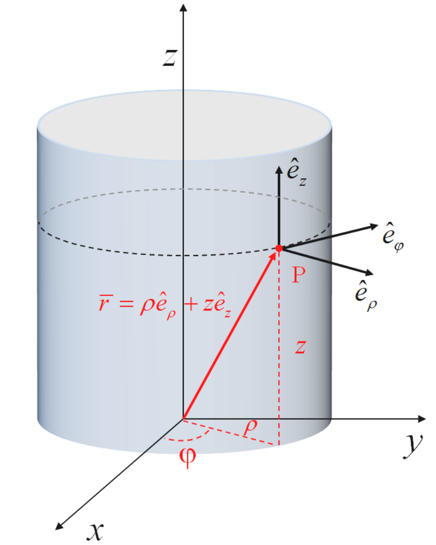
KROKLINJIGA KOORDINATER (u_1, u_2, u_3)

Basvektorer	$\hat{e}_i = \frac{1}{h_i}\frac{\partial \bar{r}}{\partial u_i}$ *med skalfaktor* $h_i = \left\lvert \frac{\partial \bar{r}}{\partial u_i} \right\rvert$
Ortsvektorns differential	$d\bar{r} = \sum_{i=1}^{3} h_i du_i \hat{e}_i$
Ytelement	$dS_3 = h_1 h_2 du_1 du_2$ (dS_3 ytan vinkelrät mot u_3-axeln)
Volymelement	$dV = h_1 h_2 h_3 du_1 du_2 du_3$
Gradienten	$\nabla\phi = \sum_i \frac{1}{h_i}\frac{\partial \phi}{\partial u_i}\hat{e}_i$
Divergensen	$\nabla\cdot\bar{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(A_1 h_2 h_3) + \frac{\partial}{\partial u_2}(A_2 h_3 h_1) + \frac{\partial}{\partial u_3}(A_3 h_1 h_2)\right]$
Rotationen	$\nabla\times\bar{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$

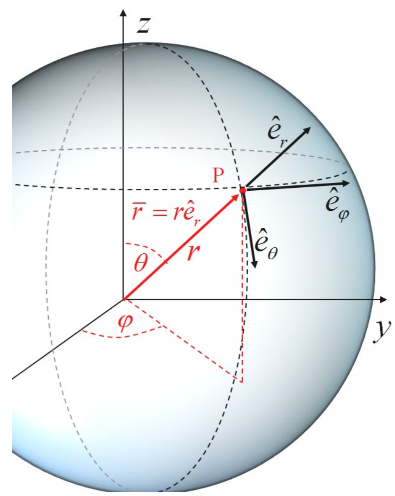
CYLINDRISKT KOORDINATSYSTEM



CYLINDRISKT KOORDINATSYSTEM

Ortsvektorn	$\bar{r} = (\rho\cos\varphi, \rho\sin\varphi, z) = \rho\cos\varphi\,\hat{e}_x + \rho\sin\varphi\,\hat{e}_y + z\,\hat{e}_z$ $\bar{r} = \rho\,\hat{e}_\rho + z\,\hat{e}_z$
Skalfaktorer	$h_\rho = 1, \quad h_\varphi = \rho, \quad h_z = 1$
Basvektorer	$\hat{e}_\rho = (\cos\varphi, \sin\varphi, 0) = \cos\varphi\,\hat{e}_x + \sin\varphi\,\hat{e}_y$ $\hat{e}_\varphi = (-\sin\varphi, \cos\varphi, 0) = -\sin\varphi\,\hat{e}_x + \cos\varphi\,\hat{e}_y$ $\hat{e}_z = (0,0,1)$
Ytelement	$dS_\rho = \rho d\varphi dz$ $dS_z = \rho d\varphi d\rho$
Volymelement	$dV = \rho d\varphi d\rho dz$
Gradienten	$\nabla\phi = \left(\frac{\partial\phi}{\partial\rho}, \frac{1}{\rho}\frac{\partial\phi}{\partial\varphi}, \frac{\partial\phi}{\partial z}\right) = \frac{\partial\phi}{\partial\rho}\hat{e}_\rho + \frac{1}{\rho}\frac{\partial\phi}{\partial\varphi}\hat{e}_\varphi + \frac{\partial\phi}{\partial z}\hat{e}_z$
Divergencen	$\nabla\cdot\bar{A} = \left(\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}\right)$
Rotationen	$\nabla\times\bar{A} = \left(\frac{\partial A_z}{\rho\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{e}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\hat{e}_\varphi + \left(\frac{\partial(\rho A_\varphi)}{\rho\partial\rho} - \frac{\partial A_\rho}{\rho\partial\varphi}\right)\hat{e}_z$
Laplaceoperatorn	$\nabla^2\phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$

SFÄRISKT KOORDINATSYSTEM



SFÄRISKT KOORDINATSYSTEM

Ortsvektorn	$\bar{r} = (r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta) = r\sin\theta\cos\varphi\,\hat{e}_x + r\sin\theta\sin\varphi\,\hat{e}_y + r\cos\theta\,\hat{e}_z$ $\bar{r} = r\,\hat{e}_r$
Skalfaktorer	$h_r = 1, \quad h_\theta = r, \quad h_\varphi = r\sin\theta$
Basvektorer	$\hat{e}_r = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) = (\sin\theta\cos\varphi)\hat{e}_x + (\sin\theta\sin\varphi)\hat{e}_y + \cos\theta\,\hat{e}_z$ $\hat{e}_\theta = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta) = (\cos\theta\cos\varphi)\hat{e}_x + (\cos\theta\sin\varphi)\hat{e}_y - \sin\theta\,\hat{e}_z$ $\hat{e}_\varphi = (-\sin\varphi, \cos\varphi, 0) = -\sin\varphi\,\hat{e}_x + \cos\varphi\,\hat{e}_y$
Ytelement	$dS_r = r^2\sin\theta d\theta d\varphi$
Volymelement	$dV = r^2\sin\theta d\theta d\varphi dr$
Gradienten	$\nabla\phi = \left(\frac{\partial\phi}{\partial r}, \frac{1}{r}\frac{\partial\phi}{\partial\theta}, \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}\right) = \frac{\partial\phi}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}\hat{e}_\varphi$
Divergencen	$\nabla\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(A_\varphi)$
Rotationen	$\nabla\times\bar{A} = \left(\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\varphi) - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial\varphi}\right)\hat{e}_r + \left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)\right)\hat{e}_\theta + \left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial\theta}\right)\hat{e}_\varphi$
Laplaceoperatorn	$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}$

L. Frassinetti

VEKTORALGEBRA FORMLER, NABLA OCH INDEXRÄKNING

$\overline{a} = (a_x, a_y, a_z)$ i kartesiska koordinater

$\overline{b} = (b_x, b_y, b_z)$ i kartesiska koordinater

$\hat{n} = (n_x, n_y, n_z)$ i kartesiska koordinater

$|\hat{n}| = 1$

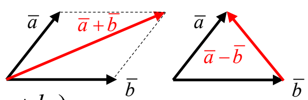
$\overline{a} = \overline{b}$ innebär att $a_x = b_x$ cykl., dvs vektorerna $\overline{a}$ och $\overline{b}$ sammanfaller efter parallell förflyttning.

$|\overline{a}| = \sqrt{\overline{a} \cdot \overline{a}} = \sqrt{a_x^2 + a_y^2 + a_z^2}$ är beloppet (dvs längden) av vektorn $\overline{a}$.

Kartesiska enhetsvektorer:

$\hat{e}_x = (1,0,0)$ $\hat{e}_y = (0,1,0)$ $\hat{e}_z = (0,0,1)$

$\overline{a} = a_x\hat{e}_x + a_y\hat{e}_y + a_z\hat{e}_z$



$\overline{a} + \overline{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$

$\overline{a} - \overline{b} = \overline{a} + \left(-\overline{b}\right) = (a_x - b_x, a_y - b_y, a_z - b_z)$

$c\overline{a} = (ca_x, ca_y, ca_z)$

$\overline{a} + \overline{b} = \overline{b} + \overline{a}$

$\left(\overline{a} + \overline{b}\right) + \overline{c} = \overline{a} + \left(\overline{b} + \overline{c}\right)$

$c\left(\overline{a} + \overline{b}\right) = c\overline{a} + c\overline{b}$

$(s+t)\overline{a} = s\overline{a} + t\overline{a}$

Skalärprodukten:

$\overline{a} \cdot \overline{b} = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$

$\overline{a} \cdot \overline{b} = |a||b|\cos\alpha \quad \Rightarrow \quad \cos\alpha = \dfrac{\overline{a} \cdot \overline{b}}{|a||b|}$

$\overline{a} \cdot \overline{b} = \overline{b} \cdot \overline{a}$

$\left(s\overline{a}\right) \cdot \left(t\overline{b}\right) = st\left(\overline{a} \cdot \overline{b}\right)$

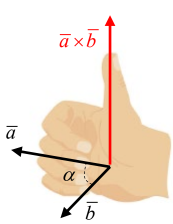
$\overline{a} \cdot \left(\overline{b} + \overline{c}\right) = \overline{a} \cdot \overline{b} + \overline{a} \cdot \overline{c}$

$\overline{a} \cdot \overline{a} = |\overline{a}|^2$

$\overline{a} \cdot \overline{b} = 0 \Leftrightarrow \overline{a} \perp \overline{b}$

Kryssprodukten

$$\overline{a} \times \overline{b} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



$\left|\overline{a} \times \overline{b}\right| = |\overline{a}|\left|\overline{b}\right|\sin\alpha$

Vektorn $\overline{a} \times \overline{b}$ är vinkelrät mot $\overline{a}$ och $\overline{b}$

$\overline{a} \times \overline{b} = -\overline{b} \times \overline{a}$

$\left(s\overline{a}\right) \times \left(t\overline{b}\right) = st\left(\overline{a} \times \overline{b}\right)$

$\overline{a} \times \left(\overline{b} + \overline{c}\right) = \overline{a} \times \overline{b} + \overline{a} \times \overline{c}$

$\overline{a} \times \overline{a} = 0$

$\hat{e}_x \times \hat{e}_y = \hat{e}_z$ $\hat{e}_y \times \hat{e}_z = \hat{e}_x$ $\hat{e}_z \times \hat{e}_x = \hat{e}_y$

Trippelprodukter: $\overline{a} \cdot \left(\overline{b} \times \overline{c}\right) = \overline{b} \cdot \left(\overline{c} \times \overline{a}\right) = \overline{c} \cdot \left(\overline{a} \times \overline{b}\right)$

$$\overline{a} \cdot \left(\overline{b} \times \overline{c}\right) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

bac-cab regeln: $\overline{a} \times \left(\overline{b} \times \overline{c}\right) = \overline{b}\left(\overline{a} \cdot \overline{c}\right) - \overline{c}\left(\overline{a} \cdot \overline{b}\right)$

$\left(\overline{a} \times \overline{b}\right) \times \overline{c} = \left(\overline{a} \cdot \overline{c}\right)\overline{b} - \left(\overline{b} \cdot \overline{c}\right)\overline{a}$

Derivator i vektoranalysen:

Här är $\overline{A}$ och $\overline{B}$ vektorfält, medan Φ och Ψ är skalärfält, dvs funktioner av läget i rummet $\overline{r} = (x, y, z)$.

$\nabla = \left(\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z}\right) = \dfrac{\partial}{\partial x}\hat{e}_x + \dfrac{\partial}{\partial y}\hat{e}_y + \dfrac{\partial}{\partial z}\hat{e}_z$ *i kartesiska koordinater*

Linjäritet :

$\nabla\left(t\Phi + s\Psi\right) = t\nabla\Phi + s\nabla\Psi$

$\nabla \cdot \left(t\overline{A} + s\overline{B}\right) = t\left(\nabla \cdot \overline{A}\right) + s\left(\nabla \cdot \overline{B}\right)$

$\nabla \times \left(t\overline{A} + s\overline{B}\right) = t\left(\nabla \times \overline{A}\right) + s\left(\nabla \times \overline{B}\right)$

Produktregler:

$\nabla\left(\Phi\Psi\right) = \Phi\nabla\Psi + \Psi\nabla\Phi$

$\nabla\left(\overline{A} \cdot \overline{B}\right) = \overline{A} \times \left(\nabla \times \overline{B}\right) + \overline{B} \times \left(\nabla \times \overline{A}\right) + \left(\overline{A} \cdot \nabla\right)\overline{B} + \left(\overline{B} \cdot \nabla\right)\overline{A}$

$\nabla \cdot \left(\Phi\overline{A}\right) = \Phi\left(\nabla \cdot \overline{A}\right) + \overline{A} \cdot \left(\nabla\Phi\right)$

$\nabla \cdot \left(\overline{A} \times \overline{B}\right) = \overline{B} \cdot \left(\nabla \times \overline{A}\right) - \overline{A} \cdot \left(\nabla \times \overline{B}\right)$

$\nabla \times \left(\Phi\overline{A}\right) = \Phi\left(\nabla \times \overline{A}\right) - \overline{A} \times \left(\nabla\Phi\right)$

$\nabla \times \left(\overline{A} \times \overline{B}\right) = \left(\overline{B} \cdot \nabla\right)\overline{A} - \left(\overline{A} \cdot \nabla\right)\overline{B} + \overline{A}\left(\nabla \cdot \overline{B}\right) - \overline{B}\left(\nabla \cdot \overline{A}\right)$

Andraderivator

$\nabla \cdot \left(\nabla \times \overline{A}\right) = 0$

$\nabla \times \left(\nabla\Phi\right) = 0$

$\nabla \times \left(\nabla \times \overline{A}\right) = \nabla\left(\nabla \cdot \overline{A}\right) - \nabla^2\overline{A}$

Indexräkning

$\overline{a} \cdot \overline{b} = a_i b_i$

$\left(\overline{a} \times \overline{b}\right)_i = \varepsilon_{ijk} a_j b_k$

$\varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{kij}$ och $\varepsilon_{ijk} = -\varepsilon_{jik}$

$\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & annars \end{cases}$ och $\begin{cases} \delta_{ii} = 3 \\ \delta_{km} l_{jm} = l_{jk} \end{cases}$

$\left(\nabla\phi\right)_i = \phi_{,i}$ $\nabla \cdot \overline{A} = A_{i,i}$ $\left(\nabla \times \overline{A}\right)_i = \varepsilon_{ijk} A_{k,j}$

Punktkälla: $\begin{cases} \overline{A}(\overline{r}) = \dfrac{s}{r^2}\hat{e}_r \text{ eller } \overline{A}(\overline{r}) = s\dfrac{\overline{r} - \overline{r}_0}{\left|\overline{r} - \overline{r}_0\right|^3}, \text{ källan i } \overline{r}_0 \\ \overline{A} = grad\,\phi \quad \Rightarrow \quad \phi = -\dfrac{s}{r} + c \\ \displaystyle\oiint_S \dfrac{s}{r^2}\hat{e}_r \cdot d\overline{S} = \begin{cases} 0 & \text{om källan yttre punkt i V} \\ 4\pi s & \text{om källan inre punkt i V} \end{cases} \end{cases}$

Dipol: $\phi = \dfrac{\overline{p} \cdot \overline{r}}{r^3}$ och $\overline{E}\left(\overline{r}\right) = -\dfrac{\overline{p}}{r^3} + \dfrac{3\left(\overline{p} \cdot \overline{r}\right)\overline{r}}{r^5}$

Virveltråd: $\displaystyle\oint_L \dfrac{k}{\rho}\hat{e}_\varphi \cdot d\overline{r} = 2\pi kN$

L. Frassinetti