

EXAMINATION SH2314 Formula Sheet

$$f(x) \approx f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \dots$$

TABLE 2.1

Basic Fourier Transform Pairs

Signal	Fourier Transform
1	$\delta(u, v)$
$\delta(x, y)$	1
$\delta(x - x_0, y - y_0)$	$e^{-j2\pi(ux_0+vy_0)}$
$\delta_s(x, y; \Delta x, \Delta y)$	comb($u\Delta x, v\Delta y$)
$e^{j2\pi(u_0x+v_0y)}$	$\delta(u - u_0, v - v_0)$
$\sin[2\pi(u_0x + v_0y)]$	$\frac{1}{2j}[\delta(u - u_0, v - v_0) - \delta(u + u_0, v + v_0)]$
$\cos[2\pi(u_0x + v_0y)]$	$\frac{1}{2}[\delta(u - u_0, v - v_0) + \delta(u + u_0, v + v_0)]$
rect(x, y)	sinc(u, v)
sinc(x, y)	rect(u, v)
comb(x, y)	comb(u, v)
$e^{-\pi(x^2+y^2)}$	$e^{-\pi(u^2+v^2)}$

TABLE 2.2

Properties of the Fourier Transform

Property	Signal	Fourier Transform
	$f(x, y)$	$F(u, v)$
	$g(x, y)$	$G(u, v)$
Linearity	$a_1f(x, y) + a_2g(x, y)$	$a_1F(u, v) + a_2G(u, v)$
Translation	$f(x - x_0, y - y_0)$	$F(u, v)e^{-j2\pi(ux_0+vy_0)}$
Conjugation	$f^*(x, y)$	$F^*(-u, -v)$
Conjugate symmetry	$f(x, y)$ is real-valued	$F(u, v) = F^*(-u, -v)$
		$F_R(u, v) = F_R(-u, -v)$
		$F_I(u, v) = -F_I(-u, -v)$
		$ F(u, v) = F(-u, -v) $
		$\angle F(u, v) = -\angle F(-u, -v)$
Signal reversing	$f(-x, -y)$	$F(-u, -v)$
Scaling	$f(ax, by)$	$\frac{1}{ ab }F\left(\frac{u}{a}, \frac{v}{b}\right)$
Rotation	$f(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$	$F(u \cos \theta - v \sin \theta, u \sin \theta + v \cos \theta)$
Circular symmetry	$f(x, y)$ is circularly symmetric	$F(u, v)$ is circularly symmetric
		$ F(u, v) = F(u, v)$
		$\angle F(u, v) = 0$
Convolution	$f(x, y) * g(x, y)$	$F(u, v)G(u, v)$
Product	$f(x, y)g(x, y)$	$F(u, v) * G(u, v)$
Separable product	$f(x)g(y)$	$F(u)G(v)$
Parseval's theorem		$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) ^2 dx dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) ^2 du dv.$