

# Causal Broadcast

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# Motivation

- Assume we have a chat application
  - Whatever written is **reliably broadcast** to group
- If you get the following output, is it ok?

**[Paris] Are you sure, the lecture is not in room B?**  
**[Lars] Room C at Electrum**  
**[Cosmin] Does anyone know where is the lecture today?**

- Cosmin's message **caused** Lars's message,
  - Lars's message **caused** Paris's message

# Motivation (2)

- Does uniform reliable broadcast remedy this?  
[d]

# Motivation (3)

- Causal reliable broadcast solves this
  - Deliveries in **causal order!**
- Causality is same as happened-before relation by Lamport!

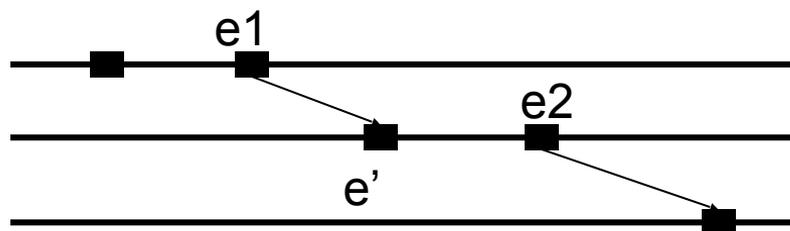
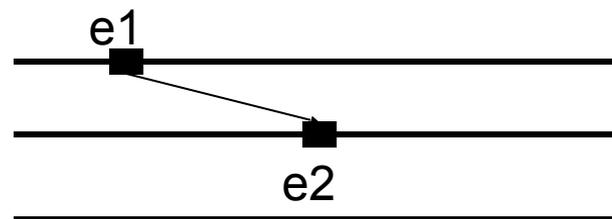
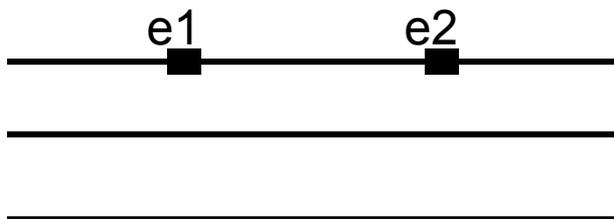
# Cause-effect relations in message passing systems

- An event  $e_1$  may potentially have caused another event  $e_2$  if the following relation, called, *happens-before* and denoted by  $e_1 \rightarrow e_2$  holds

# Happens-before relation

- $e_1$  and  $e_2$  occurs at the same process  $p$ , and  $e_1$  occurs before  $e_2$
- $e_1$  is the transmission of a message  $m$  at process  $p$  and  $e_2$  is the reception of the same message at process  $q$
- There exist some event  $e'$  such that  $e_1 \rightarrow e'$  and  $e' \rightarrow e_2$

# Happens-before relation



# Intuitions (1)

- So far, we did not consider ordering among messages; In particular, we considered messages to be independent
- Two messages from the same process might not be delivered in the order they were broadcast
- A message  $m_1$  that causes a message  $m_2$  might be delivered by some process after  $m_2$

# Intuitions (2)

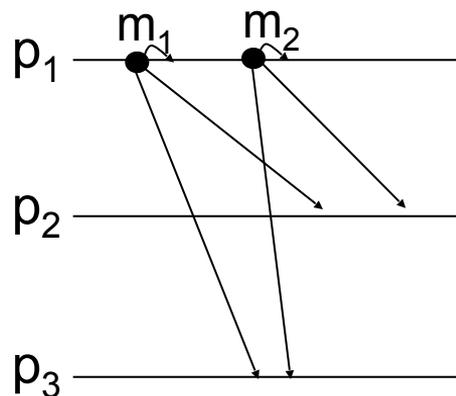
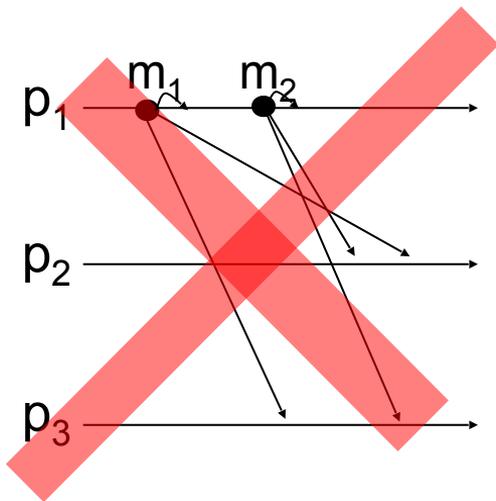
- Causal broadcast means
  - Causality between broadcast events is preserved by the corresponding delivery events
  - If  $\text{broadcast}(m1)$  happens-before  $\text{broadcast}(m2)$ , any  $\text{delivery}(m2)$  cannot happen-before a  $\text{delivery}(m1)$

# Causality of Messages

- Let  $m_1$  and  $m_2$  be any two messages:  
 $m_1 \rightarrow m_2$  ( $m_1$  **causally precedes**  $m_2$ ) if
  - **C1 (FIFO order).**
    - Some process  $p_i$  broadcasts  $m_1$  before broadcasting  $m_2$
  - **C2 (Network order).**
    - Some process  $p_i$  delivers  $m_1$  and later broadcasts  $m_2$
  - **C3 (Transitivity).**
    - There is a message  $m'$  such that  $m_1 \rightarrow m'$  and  $m' \rightarrow m_2$

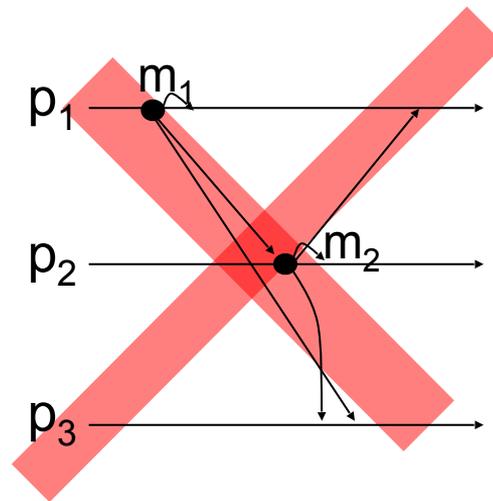
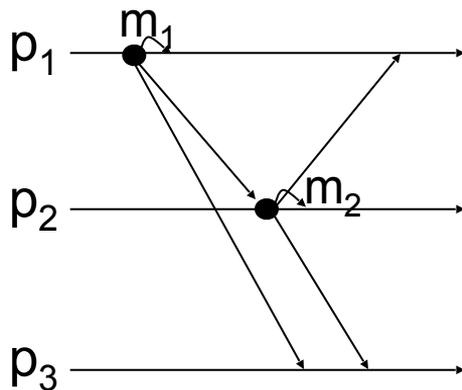
# Causality

- **C1 (FIFO order).**
  - Some process  $p_i$  broadcasts  $m_1$  before broadcasting  $m_2$



# Causality (2)

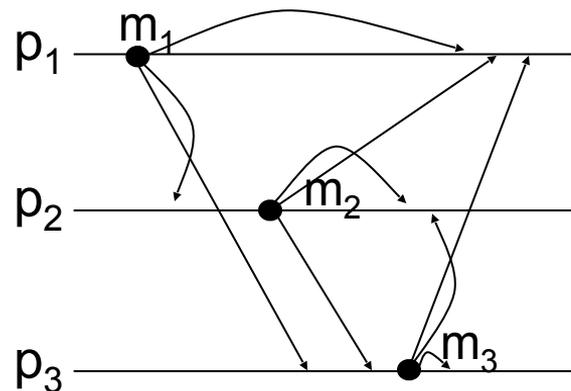
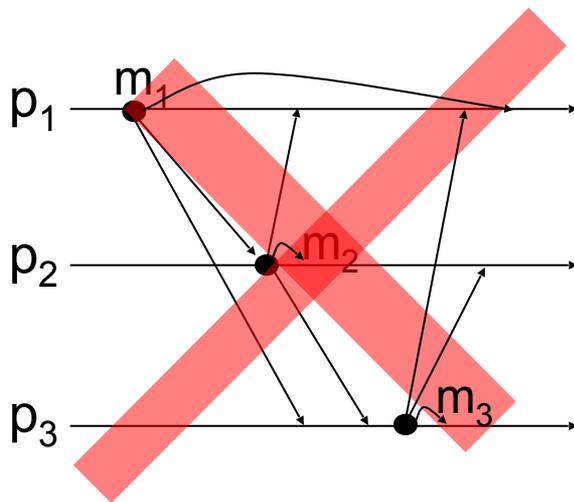
- **C2 (Network order).**
  - Some process  $p_i$  delivers  $m_1$  and later broadcasts  $m_2$



# Causality (3)

- **C3 (Transitivity).**

- There is a message  $m'$  such that  $m_1 \rightarrow m'$  and  $m' \rightarrow m_2$



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# **Specification of causal reliable broadcast**

# Causal Broadcast Interface

- **Module:**

- Name: CausalOrder (co)

- **Events**

- Request:  $\langle \text{co Broadcast} \mid m \rangle$
- Indication:  $\langle \text{co Deliver} \mid \text{src}, m \rangle$

- **Property:**

- **CB:** If node  $p_i$  delivers  $m_1$ , then  $p_i$  must have delivered every message causally preceding ( $\rightarrow$ )  $m_1$  before  $m_1$

# Causal Broadcast Interface

- If node  $p_i$  delivers  $m_1$ , then  $p_i$  must have delivered every message causally preceding ( $\rightarrow$ )  $m_1$  before  $m_1$
- Is this useful? How can it be satisfied? [d]
  - It is only safety. Satisfy it by never delivering!

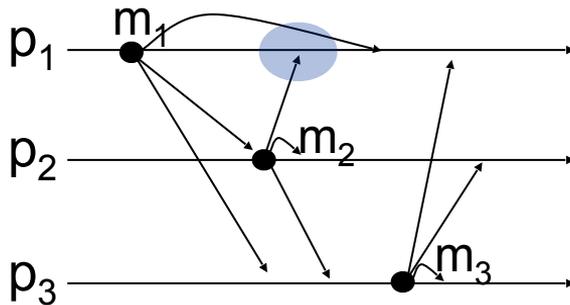
# Different Causalities

- **Property:**
  - **CB:** If node  $p_i$  delivers  $m_1$ , then  $p_i$  must deliver every message causally preceding ( $\rightarrow$ )  $m_1$  before  $m_1$
  - **CB':** If  $p_i$  delivers  $m_1$  and  $m_2$ , and  $m_1 \rightarrow m_2$ , then  $p_i$  must deliver  $m_1$  before  $m_2$
- What is the difference? [d]

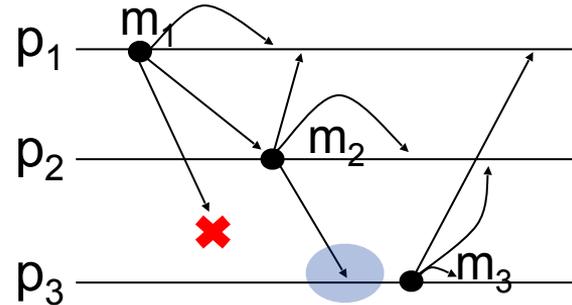
# Different Causalities

- **Property:**
  - **CB:** If node  $p_i$  delivers  $m_1$ , then  $p_i$  must deliver every message causally preceding ( $\rightarrow$ )  $m_1$  before  $m_1$
  - **CB':** If  $p_j$  delivers  $m_1$  and  $m_2$ , and  $m_1 \rightarrow m_2$ , then  $p_j$  must deliver  $m_1$  before  $m_2$
- What is the difference? [d]

Violates CB and CB'



Violates CB, not CB'



- Indeed, CB implies CB'

# Reliable Causal Broadcast Interface

- **Module:**
  - Name: `ReliableCausalOrder (rco)`
- **Events**
  - Request: `⟨rco Broadcast | m⟩`
  - Indication: `⟨rco Deliver | src, m⟩`
- **Property:**
  - **RB1-RB4** from regular reliable broadcast
  - **CB:** If node  $p_i$  delivers  $m$ , then  $p_i$  must deliver every message causally preceding ( $\rightarrow$ )  $m$  before  $m$

# Uniform Reliable Causal Broadcast

- **Module:**
  - Name: `UniformReliableCausalOrder` (urco)
- **Events**
  - Request: `<urco Broadcast | m>`
  - Indication: `<urco Deliver | src, m>`
- **Property:**
  - **URB1-URB4** from uniform reliable broadcast
  - **CB:** If node  $p_i$  delivers  $m$ , then  $p_i$  must deliver every message causally preceding ( $\rightarrow$ )  $m$  before  $m$

# Idea reuse...

- Reuse RB for CB
- Use **reliable broadcast** abstraction to implement **reliable causal broadcast**
- Use **uniform reliable broadcast** abstraction to implement **uniform causal broadcast**

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# **Implementation of causal reliable broadcast**

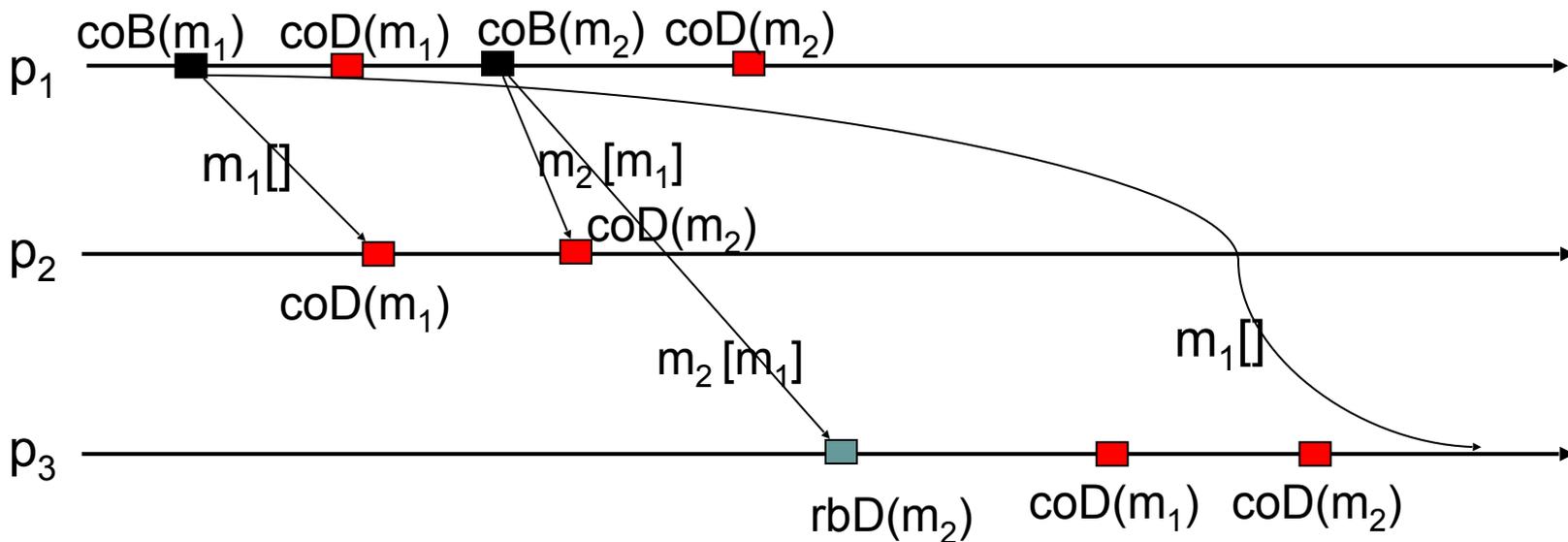
# Towards an implementation

- Main idea
  - Each broadcasted message carries a **history**
  - Before delivery, ensure causality
- First algorithm
  - History is set of all **causally preceding** messages

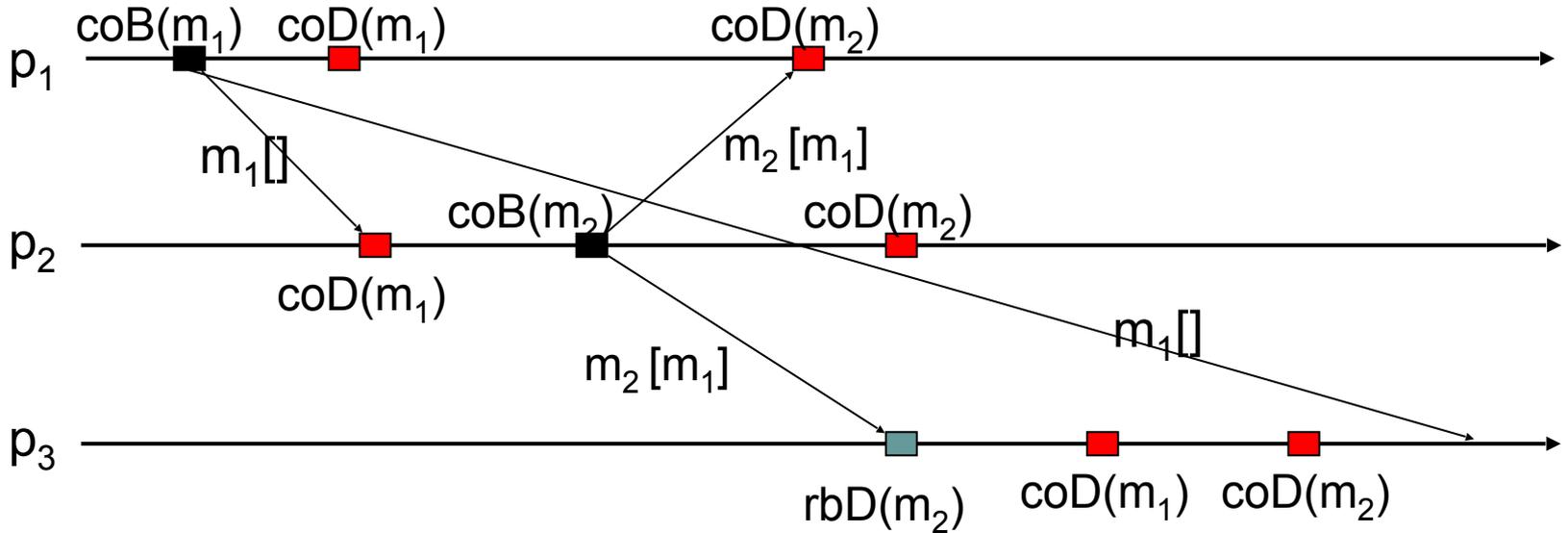
# Fail-Silent No-Waiting Causal Broadcast

- Each message  $m$  carries **ordered list** of causally preceding messages in **past <sub>$m$</sub>**
- Whenever a node  $rb$ -Delivers  $m$ 
  - co-Deliver causally preceding messages in **past <sub>$m$</sub>**
  - co-Delivers  $m$ 
    - Avoid duplicates using **delivered**

# Execution (direct override)



# Execution (indirect override)



# Fail-silent Causal Broadcast Impl

- **Implements:**
  - ReliableCausalOrderBroadcast (rco)
- **Uses:** ReliableBroadcast (rb)
- **upon event**  $\langle \text{Init} \rangle$  **do**
  - $\text{delivered} := \emptyset$ ;  $\text{past} := \text{nil}$
- **upon event**  $\langle \text{rco Broadcast} \mid m \rangle$  **do**
  - **trigger**  $\langle \text{rb Broadcast} \mid (\text{DATA}, \text{past}, m) \rangle$
  - $\text{past} := \text{append}(\text{past}, (p_i, m))$

**Append this  
message to past  
history**

# Fail-silent Causal Broadcast Impl (2)

- upon event  $\langle \text{rb Deliver} \mid p_i, (\text{DATA}, \text{past}_m, m) \rangle$  do

- if  $m \notin \text{delivered}$  then

- forall  $(s_n, n) \in \text{past}_m$  do

in ascending order

- if  $n \notin \text{delivered}$  then

- trigger  $\langle \text{rco Deliver} \mid s_n, n \rangle$

deliver preceding messages

- delivered := delivered  $\cup$  {n}

- past := append(past, (s\_n, n))

append to history

- trigger  $\langle \text{rco Deliver} \mid p_i, m \rangle$

deliver current message

- delivered := delivered  $\cup$  {m}

- past := append(past, (p\_i, m))

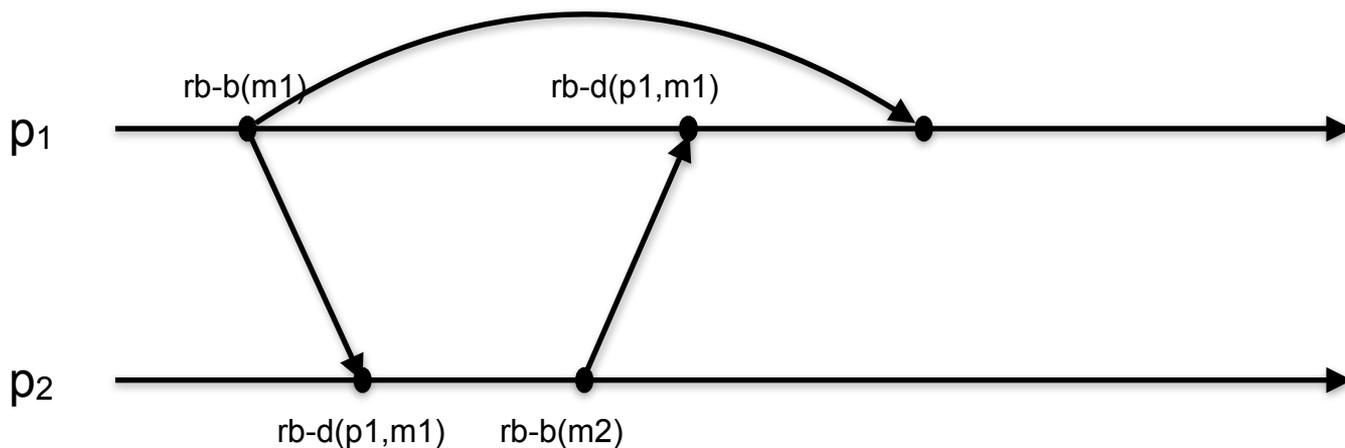
append to history

# Correctness

- RB1-RB4 follow from use of RB
  - No creation and no duplication still satisfied
  - Validity still satisfied
    - Some messages might be delivered earlier, never later
  - Agreement directly from RB

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# Correctness

- RB1-RB4 follow from use of RB
  - Agreement directly from RB
  - If correct process  $p_k$  delivers all correct processes deliver
    - all processes will deliver because of RB agreement either immediately or included in the  $\text{past}_m$  of previous message  $m$

# Correctness of CB

- If process  $p_i$  delivers  $m$ , then  $p_i$  must deliver every message causally preceding ( $\rightarrow$ )  $m$  before  $m$
- This property is an **invariant** of each execution (or prefix of)
- $P$  is an **invariant** if  $P(E)$  holds for all executions  $E$
- If  $P(E)$  is an **invariant**,
  - $P$  hold for all  $s_0$  in the set of initial states
  - If  $P$  holds in execution (prefix)  $E$  with final state  $s_n$  then  $P$  holds after extending  $E$  with any transition step  $(s_n, e_{n+1}, s_{n+1})$

# Correctness CB

- Each message carries its causal past
  - Each delivery of a message  $m$  makes sure that its causal past is delivered before  $m$
- CO by **induction** on prefixes of executions
  - It is true for empty executions (initial state  $s_0$ )
  - Assume it is true for all deliveries of a prefix
    - Then it is true for any extension with one more event

# Improving the algorithm

- Disadvantage of algorithm is that the message size (bit complexity) grows
- Useful idea
  - **Garbage collect** old messages
- Implementation of GC
  - Acknowledge causal delivery of every message  $m$  to all processes
  - Use perfect failure detector **P**
    - Determine with **P** when all correct nodes got message  $m$
    - Delete  $m$  from past when all correct processes got  $m$

# Improving the algorithm

- We use **P**
- Use FIFO reliable broadcast
- It is possible to trim **Past** ?

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# **Causal Broadcast Algorithm using FIFO Broadcast**

# Causal Broadcast Interface

- **Module:**
  - Name: `FIFO-ReliableBroadcast (frb)`
- **Events**
  - Request: `<frb Broadcast | m>`
  - Indication: `<frb Deliver | src, m>`
- **Property:**
  - ***FIFO delivery***: if  $p_i$  broadcasts message  $m_1$  before it broadcasts message  $m_2$ , then no correct process delivers  $m_2$  unless it has already delivered  $m_1$
  - ***RB1-RB4***

# Idea of using FIFO reliable broadcast

- Assume we use fifo-rb instead rb
- In the no-waiting algorithm
  - Each process  $p_i$  rb-broadcasts the message  $\text{append}(\text{past}_m, m)$
  - Assume two consecutive broadcasts by  $p_i$ 
    - $\text{append}(\text{past}_{m1}, m_1)=l_1$  and then  $\text{append}(\text{past}_{m2}, m_2)=l_2$
    - Each correct process delivers  $l_1$  before  $l_2$  by FIFO delivery
    - But  $l_1$  is a prefix of  $l_2$  so  $p_i$  needs to only broadcast  $l_2 - l_1$
  - Each  $p_i$  needs to keep track only of messages between to consecutive broadcasts

# Fail-silent Causal Broadcast Impl

- **Implements:**
    - ReliableCausalOrderBroadcast (rco)
  - **Uses:** FIFO-ReliableBroadcast (frb)
  - **upon event**  $\langle \text{Init} \rangle$  **do**
    - $\text{delivered} := \emptyset; l := \text{nil}$
  - **upon event**  $\langle \text{rco Broadcast} \mid m \rangle$  **do**
    - **trigger**  $\langle \text{frb Broadcast} \mid (\text{DATA}, \text{append}(l, m)) \rangle$
    - $l := \text{nil}$
- reset  $l$  to store only  
new deliveries**
- 
- A red arrow points from the box above to the
- $l := \text{nil}$
- line in the list.

# Fail-silent Causal Broadcast Impl (2)

- upon event  $\langle \text{frb Deliver} \mid \text{pi}, (\text{DATA}, l_m) \rangle$  do

- forall  $(s_n, n) \in l_m$  do

- if  $n \notin \text{delivered}$  then

← in ascending order

- trigger  $\langle \text{rco Deliver} \mid s_n, n \rangle$

← deliver message

- $\text{delivered} := \text{delivered} \cup \{n\}$

- if  $(s_n, n) \notin l$  then

- append( $l, (s_n, n)$ )

← append to local  $l$

- Can we trim the **delivered** set? [d]

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# Fail-Silent Waiting Algorithm

# Towards another implementation

- Main idea
  - Each broadcasted message carries a **history**
  - Before delivery, ensure causality
- First & Second algorithms
  - History is set of all **causally preceding** messages
- Third algorithm [d]
  - History is a **vector timestamp**

# Fail-Silent Waiting Causal Broadcast

- Represent past history by **vector clock (VC)**
- Slightly modify the VC implementation
  - At process  $p_i$ 
    - $VC[i]$ : number of messages  $p_i$  coBroadcasted
    - $VC[j]$ ,  $j \neq i$ : number of messages  $p_i$  coDelivered from  $p_j$

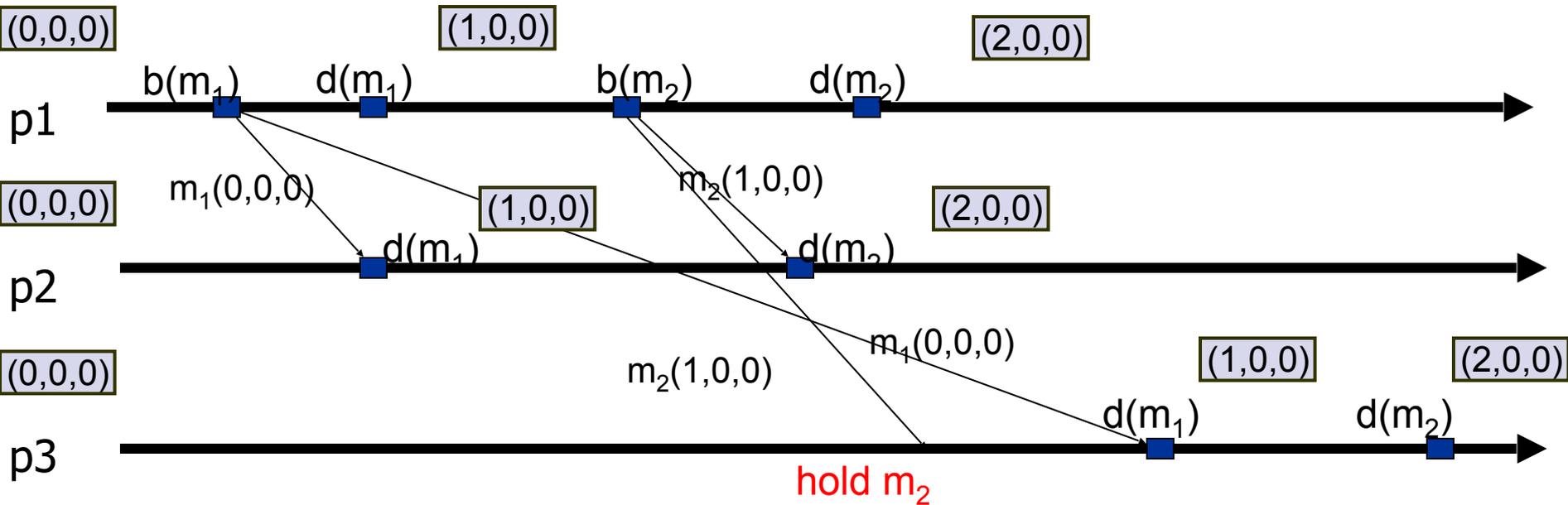
# Fail-Silent Waiting Causal Broadcast

- Upon CO broadcast  $m$ 
  - Piggyback VC and RB-broadcast  $m$
  - $VC_m[r]$  is the number messages causally preceding  $m$  from  $r$
- Upon RB delivery of  $m$  with attached  $VC_m$   
compare  $VC_m$  with local  $VC_i$ 
  - Only deliver  $m$  once  $VC_m \leq VC_i$
  - **Do Not deliver** if  $VC_m > VC_i$  or  $VC_m \neq VC_i$

# Fail-Silent Waiting Causal Broadcast

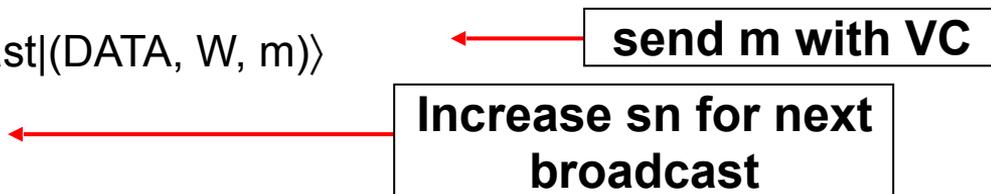
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  - Only deliver  $m$  once  $VC_m \leq VC_i$
  - **Do Not deliver** if  $VC_m > VC_i$  or  $VC_m \neq VC_i$

# Execution



# Fail-Silent Waiting Causal Implementation

- **Uses:** ReliableBroadcast (rb)
- **upon event**  $\langle \text{Init} \rangle$  **do**
  - **forall**  $p_i \in \Pi$  **do**  $\text{VC}[p_i] := 0$
  - $\text{sn} := 0$
  - $\text{Pending} := \emptyset$
- **upon event**  $\langle \text{rc Broadcast} | m \rangle$  **do**
  - $W = \text{copy}(\text{VC})$
  - $W[\text{self}] := \text{sn}$
  - **trigger**  $\langle \text{rb Broadcast} | (\text{DATA}, W, m) \rangle$
  - $\text{sn} := \text{sn} + 1$



# Fail-Silent Waiting Causal Impl. (2)

- **upon event**  $\langle \text{rbDeliver} | p_j, (\text{DATA}, \text{VC}_m, m) \rangle$  **do**

- $\text{pending} := \text{pending} \cup (p_j, (\text{DATA}, \text{VC}_m, m))$  ← **put on hold**
- $\text{deliver-pending}()$

**for every message  
whose VC precedes  
local VC**

- **proc**  $\text{deliver-pending}()$

- **while exists**  $x=(s_m, (\text{DATA}, \text{VC}_m, m)) \in \text{pending}$  s.t.  $\text{VC}_m \leq \text{VC}$  **do**
  - $\text{pending} := \text{pending} \setminus (s_m, (\text{DATA}, \text{VC}_m, m))$
  - $\text{VC}[s_m] := \text{VC}[s_m] + 1$
  - **trigger**  $\langle \text{rcoDeliver} | s_m, m \rangle$

**Remove on hold  
deliver and increase  
local VC**

# Correctness

- **Validity**

- $m$  is co-cast by a correct  $p_i$  with  $VC_m$  equal  $VC_i$  at **send time** or higher only at  $VC_i[i]$  by outstanding earlier co-cast not delivered yet
- By rb-cast validity  $m$  is eventually rb-delivered at  $p_i$  as well as earlier co-casts
- At delivery time  $VC_i$  can only increase, so
- Eventually  $VC_m \leq VC_i$  and  $m$  is co-delivered

**upon event**  $\langle rco \text{ Broadcast} | m \rangle$  **do**

$W = \text{copy}(VC)$

$W[\text{self}] := sn$

**trigger**  $\langle rb\text{Broadcast} | (DATA, W, m) \rangle$

$sn := sn + 1$

# Correctness

- **Agreement**
  - Assume  $m$  is co-delivered at correct  $p_i$
  - $p_i$  co-delivered all message causally before  $m$
  - Every correct process  $r_b$ -delivered  $m$  and all causally preceding messages (agreement of RB)
  - Hence every correct process co-deliver  $m$

# Correctness

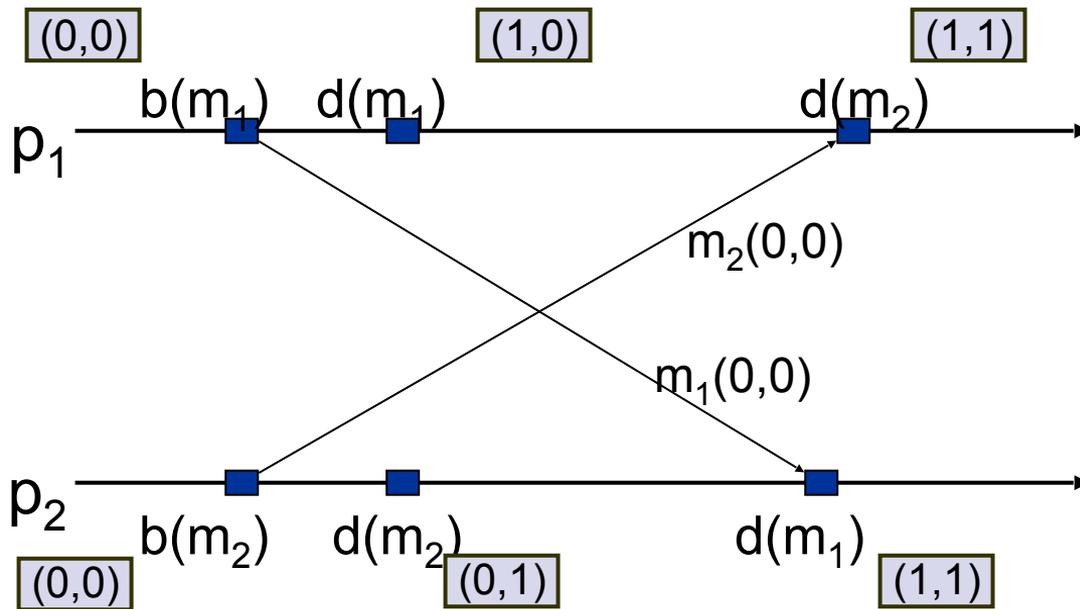
- **Causal Order**
  - Assume  $p$  rb-delivers  $m$ ,  $VC_m$  from  $q$
  - $VC_m[r]$  is the number messages causally preceding  $m$  from  $r$
  - VC at  $p$  stores the number of messages co-delivered from each process
  - For some  $r$ ,  $VC_m[r] > VC[r]$  implies there is at least one message from  $r$  that is causally before  $m$ , which is not co-delivered at  $p$
  - $P$  waits to deliver  $m$  until  $VC_m[r] \leq VC_i[r]$ , for all  $r$
  - Hence  $m$  is not delivered until all causally preceding messages are delivered

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# Orderings of Broadcast

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# Possible execution?



- Delivery order isn't same!
  - What is wrong? **[d]** Nothing, there is no causality.

# Other possible orderings

- Other common orderings
  - Single-source **FIFO order**
  - **Total order**
  - **Causal order**

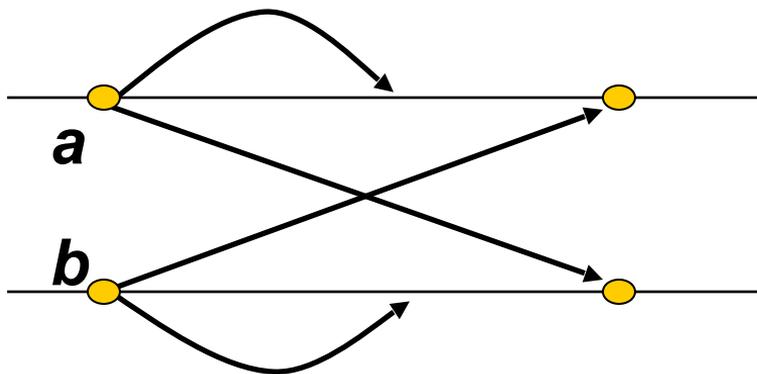
# Single-Source FIFO order

- Intuitively
  - Msgs from **same node** delivered in **order sent**
- For all messages  $m_1$  and  $m_2$  and all  $p_i$  and  $p_j$ ,
  - if  $p_i$  broadcasts  $m_1$  before  $m_2$ , and if  $p_j$  delivers  $m_2$ , then  $p_j$  delivers  $m_1$  before  $m_2$
- Caveat
  - This formulation doesn't require delivery of both messages

# Total Order

- Intuitively
  - **Everyone** delivers everything in exact **same order**
- For all messages  $m_1$  and  $m_2$  and all  $p_i$  and  $p_j$ ,
  - if both  $p_i$  and  $p_j$  deliver both messages, then they deliver them in the same order
- Caveat
  - This formulation doesn't require delivery of both messages
  - Everyone delivers same order, maybe not send order!

# Execution Example (1)

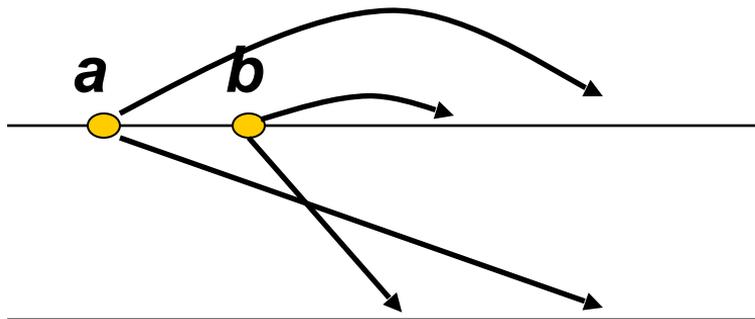


single-source FIFO? **yes**

totally ordered? **no**

causally ordered? **yes**

# Execution Example (2)

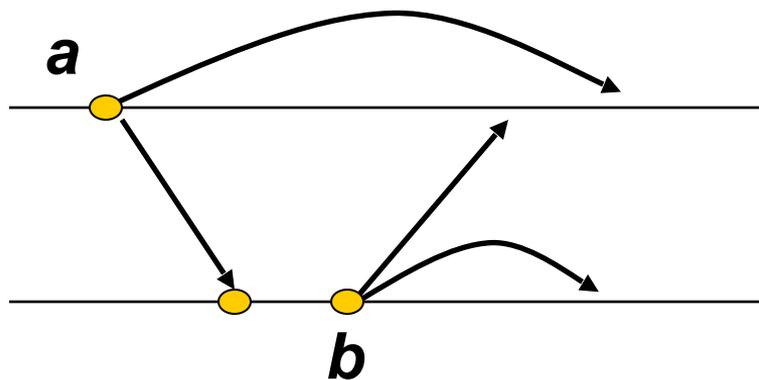


single-source FIFO? **no**

totally ordered? **yes**

causally ordered? **no**

# Execution Example (3)



single-source FIFO? **yes**

totally ordered? **no**

causally ordered? **no**

# Hierarchy of Orderings

- Stronger **implies** weaker ordering ( $\rightarrow$ )

