

Föreläsning 15

S1=1661 HT 20

9/10

Repetition

Från "Modell tenta 1"

1.

$$z = -2 - 2i, w = 2e^{i\pi/3}$$

$$|z| = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$

$$z = 2\sqrt{2} \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) =$$

$$= 2\sqrt{2} \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right) = 2\sqrt{2} e^{i5\pi/4}$$



$$\frac{w^8}{z^6} = \frac{(2e^{i\pi/3})^8}{(2^{3/2} e^{i5\pi/4})^6} = \frac{2^8 \cdot e^{i8\pi/3}}{2^9 \cdot e^{i30\pi/4}} =$$

$$\frac{2^8 \cdot e^{i8\pi/3}}{2^9 \cdot e^{i30\pi/4}} = \frac{1}{2} \frac{e^{i(2\pi + \frac{2\pi}{3})}}{e^{i(6\pi + \frac{3\pi}{2})}}$$

$6\pi + \frac{3\pi}{2} = \frac{24}{4}\pi + \frac{6}{4}\pi$

$$= \left\{ e^{iV} \text{ periodisk med period } 2\pi \right\}$$

$e^{iV} = (\cos V + i \sin V)$

$$= \frac{1}{2} \frac{e^{i2\pi/3}}{e^{i3\pi/2}} = \frac{1}{2} e^{i\left[\frac{2\pi}{3} - \frac{3\pi}{2}\right]}$$

$$\frac{1}{2} e^{i \left[\frac{2\pi}{3} - \frac{3\pi}{2} \right]} = \frac{1}{2} e^{i \left[\frac{4\pi - 9\pi}{6} \right]}$$

$$= \frac{1}{2} e^{i \left(-\frac{5\pi}{6} \right)}$$



$$= \frac{1}{2} \left(\cos \left(-\frac{5\pi}{6} \right) + i \sin \left(-\frac{5\pi}{6} \right) \right) =$$

$$= \frac{1}{2} \left(\cos \frac{5\pi}{6} - i \sin \frac{5\pi}{6} \right) = \frac{1}{2} \left(-\frac{\sqrt{3}}{2} - i \frac{1}{2} \right)$$

$$\boxed{\text{Svar: } = -\frac{\sqrt{3}}{4} - i/4}$$

$$\textcircled{1} \quad x^2 - 8x = 6y - y^2$$

$$x^2 - 8x + y^2 - 6y = 0$$

$$(x-4)^2 + (y-3)^2 = 16 + 9$$

$$x^2 - 8x + 16 \quad y^2 - 6y + 9$$

$$(x-4)^2 + (y-3)^2 = 5^2$$

Svar: Medelpkt: $(4, 3)$, $r = 5$

3. Vi använder $e^{x+y} = e^x \cdot e^y, \forall x, y \in \mathbb{R}$ (1)
för att visa $\ln r + \ln t = \ln(rt), \forall r, t \in (0, \infty)$ (2).

$$e^{\text{V.L.}(2)} = e^{\ln r + \ln t} \stackrel{(1)}{=} e^{\ln r} \cdot e^{\ln t} = rt$$

$$e^{\text{H.L.}(2)} = e^{\ln(rt)} = rt = \underline{e^{\text{V.L.}(2)}}$$

$$\ln(\underline{e^{\text{H.L.}(2)}}) = \ln(\underline{e^{\text{V.L.}(2)}}) \Rightarrow \text{H.L.}(2) = \text{V.L.}(2)$$

V.S.B.

$$4(a) \quad S = \sum_{k=1}^{1000} \frac{k}{100} = \left(\frac{1}{100} + \frac{2}{100} + \dots + \frac{1000}{100} \right)$$

$$= \frac{1}{100} (1 + 2 + \dots + 1000) = \frac{1}{100} \underbrace{\sum_{k=1}^{1000} k}_{S_1}$$

Arithmetisk:

$$S_1 = (1 + 2 + \dots + 1000)$$

$$+ \quad S_1 = (1000 + 999 + \dots + 1)$$

↑
leavis
addition

$$2 S_1 = 1001 + 1001 + \dots + 1001 = 1000 \cdot 1001$$

$$S_1 = \frac{1000 \cdot 1001}{2}, \quad S = \frac{1}{100} S_1 = \frac{1000 \cdot 1001}{2 \cdot 100} \stackrel{\text{Svar:}}{=} \underline{\underline{5005}}$$

$$\underline{Alt:} \quad \sum_{k=1}^{1000} \frac{k}{100} \quad \text{aritmetisk}$$

$$= \frac{(\text{första term} + \text{sista term})}{2} \cdot (\text{antal termer})$$

$$= \frac{\left(\frac{1}{100} + 10 \right)}{2} \cdot 1000 = \quad = \underline{\underline{5005}}$$

$$4 \text{ b) } T = \sum_{j=3}^9 2^j = (2^3 + 2^4 + \dots + 2^9)$$

geometrisk summa med kvot $q=2$

$$qT = 2T$$

$$T = 2^3 + 2^4 + \dots + 2^8 + 2^9$$

$$2T = 2^4 + 2^5 + \dots + 2^9 + 2^{10}$$

$$-T = 2^3 - 2^{10}$$

Reduis

Subtraktion

$$T = 2^{10} - 2^3 = 1024 - 8 = \underline{\underline{1016}} = \underline{\text{Svar}}$$

Alt: $\sum_{j=3}^9 2^j = \underbrace{2^3 + \dots + 2^9}_{7 \text{ termer}}$

$= \left\{ \begin{array}{l} \text{geom.} \\ \text{första term} = 2^3 = 8 \\ \text{kvot} = 2 \end{array} \right\}$

$= \text{"första term"} \cdot \frac{1 - \text{kvot}^{\text{antal termer}}}{1 - \text{kvot}} = 8 \cdot \frac{1 - 2^7}{1 - 2}$

$= 8(2^7 - 1) = 8 \cdot 127 = \underline{1016}$

5.

$$r = 1,222\overline{34} = 1,222343434\dots$$

r has periodic decimal expansion

~~is~~
SATS

$r \in \mathbb{Q}$, thus finds $n, m \in \mathbb{Z}$ s.a. $r = \frac{n}{m}$

$$r = 1,222\overline{3434\dots}$$

$$100r = 122,234\overline{3434\dots}$$

99r

$$\begin{aligned} 100r - r &= 122,234\overline{34} - 1,222\overline{34} = 122,234 - 1,222 \\ &= 121,012, \Rightarrow 99r = 121,012 \end{aligned}$$

$$99r = 121,012 \quad \Rightarrow \quad r = \frac{121,012}{99}$$

$$\Rightarrow r = \frac{121012}{99000} = \frac{30253}{24750}$$

Svar

6.

$$\left(x^3 + \frac{2}{x^2}\right)^{10}$$

a + b

Binom

Satz von k=0

$$\sum_{k=0}^{10} \binom{10}{k} (x^3)^{10-k} \left(\frac{2}{x^2}\right)^k$$

$$a^{10-k} \cdot b^k$$

$$= \sum_{k=0}^{10} \binom{10}{k} x^{30-3k} \cdot \frac{2^k}{x^{2k}} =$$

k=0

$$= \sum_{k=0}^{10} \binom{10}{k} 2^k \cdot x^{\underline{30-5k}}$$

Konstantteil
für da

$$30-5k=0$$

$$\text{dus } k=6$$

och då är

$$\binom{10}{k} \cdot 2^k = \binom{10}{6} \cdot 2^6 = \frac{10!}{6!4!} 2^6 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} \cdot 2^6$$

$$= \frac{10 \cdot \cancel{9} \cdot \cancel{8} \cdot 7}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} \cdot 2^6 = 10 \cdot 21 \cdot 64 = \dots = \underline{\underline{13440}}$$

$$\left(x^3 + \frac{2}{x^2}\right)^{10} = x^{30} + \dots + \boxed{13440} + \dots + \frac{2^{10}}{x^{20}}$$

Svar: 13440

⑦. $f(x) = \frac{x+a}{x+b}$ $D_f = \{x \in \mathbb{R}, x \neq -b\}$, $V_f = \{y \in \mathbb{R} : y \neq 1\}$

OBS 1: $D_f = \{x \in \mathbb{R} : x \neq -b\}$

OBS 2: Om $a=b$, $f(x) = \frac{x+a}{x+a} = 1 \quad \forall x \neq -a$

är f ej inverterbar

Så antag att $a \neq b$ fr.o.m. nu

Söker f^{-1} med std-metod:

$y = f(x)$ och lös att x (kolla om det går för vilken y helst)

$$\textcircled{y} \neq f(x) \quad y = \frac{x+a}{x+b} \quad \Leftrightarrow \quad (x+b|y = (x+a)) \quad x \neq -b$$

$$\Leftrightarrow xg + by = x + a \quad \Leftrightarrow \quad x(y-1) = a - by$$

$$\Leftrightarrow \textcircled{x} = \frac{a - by}{y - 1} \quad y \neq 1$$

$$\left(1 = \frac{x+a}{x+b} \quad \Leftrightarrow \quad x+b = x+a \quad \Leftrightarrow \quad a = b \right)$$

man auch $a \neq b$

$$y = f(x) \quad \Leftrightarrow \quad x = \frac{a - by}{y - 1} = f^{-1}(y), \quad \text{d.h.} \quad f^{-1}(x) = \frac{a - bx}{x - 1}$$

dus
 $\tilde{f}(x) = \frac{a-bx}{x-1}$

$\forall \tilde{f}$ über a, b s.g. $\frac{f(x)}{x-1} = \frac{f^{-1}(x)}{x-1}$
 $\text{dus } \frac{x+a}{x+b} = \frac{a-bx}{x-1} \quad \begin{matrix} x \neq -b \\ x \neq 1 \end{matrix}$

$\Leftrightarrow (x+a)(x-1) = (x+b)(a-bx)$

$\Leftrightarrow \underline{x^2 + (a-1)x - a = -bx^2 + (a-b^2)x + ab}$

$\Leftrightarrow \begin{cases} -b = 1 \\ a-b^2 = a-1 \\ ab = -a \end{cases} \Rightarrow \begin{matrix} b = -1 \\ \text{erfüllt alle } a \text{ mit } b = -1 \end{matrix} \quad \Bigg| \quad \underline{\text{Somit:}} \quad \begin{cases} b = -1 \\ a \neq -1 \end{cases}$

8.

$$|x^2 - 2x - 4| < 1 \quad (1)$$

$$\boxed{\begin{array}{l} |a| < b \\ \Leftrightarrow -b < a < b \end{array}}$$

$$\Leftrightarrow -1 < x^2 - 2x - 4 < 1$$

$$\Leftrightarrow \begin{cases} x^2 - 2x - 4 < 1 \\ x^2 - 2x - 4 > -1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2 - 2x - 5 < 0 & (2) \\ x^2 - 2x - 3 > 0 & (3) \end{cases}$$

$$\begin{cases} x^2 - 2x - 5 < 0 & (2) \\ x^2 - 2x - 3 > 0 & (3) \end{cases}$$

(2) $x^2 - 2x - 5 < 0$

Findet. v. L.

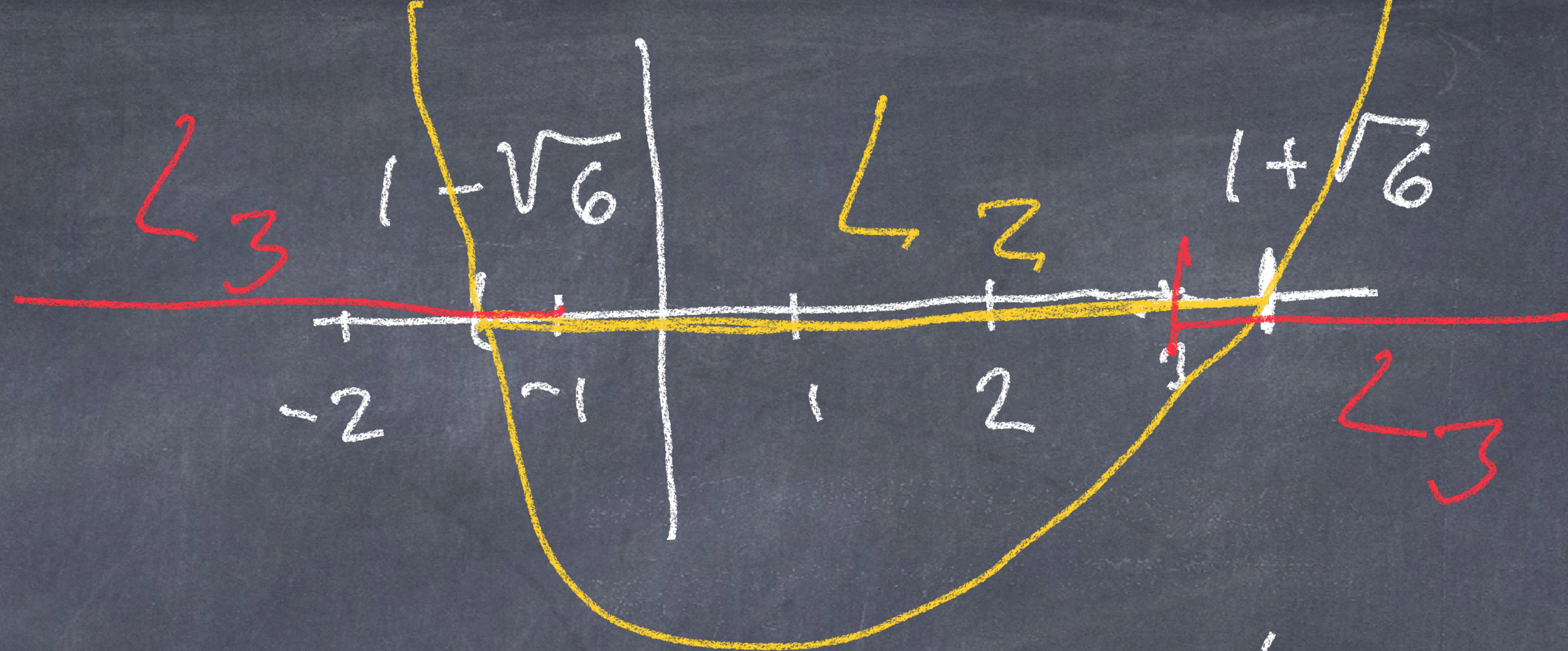
$$x^2 - 2x - 5 = 0$$

$$x = 1 - \sqrt{6} \text{ oder } x = 1 + \sqrt{6}$$

so

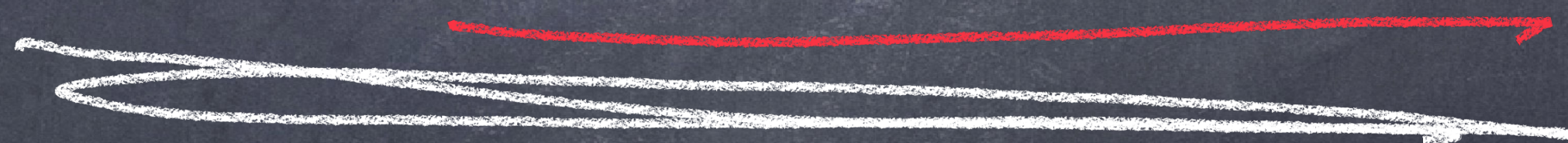
(2) $\Rightarrow$

$$(x - (1 - \sqrt{6})) (x - (1 + \sqrt{6})) < 0$$



(2) hier Lösung. ang. L

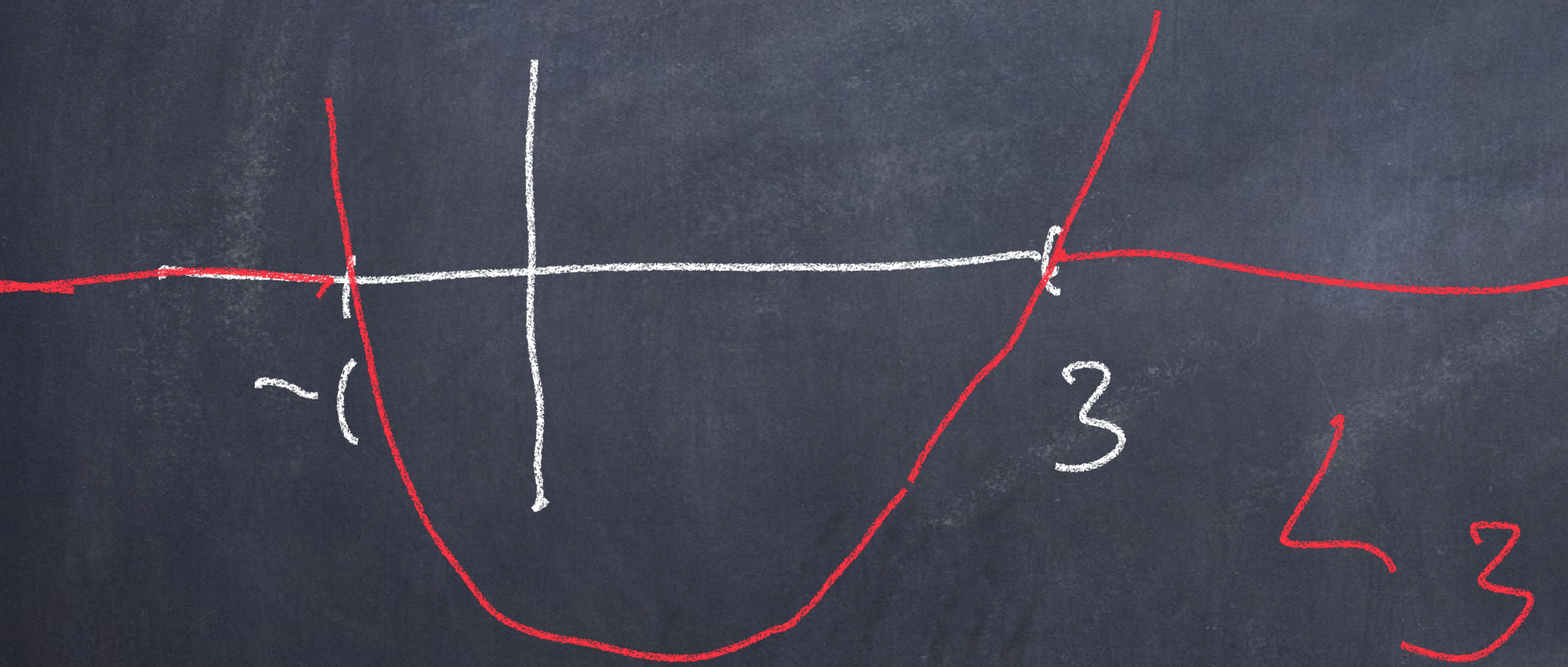
$$L_2 = (1 - \sqrt{6}, 1 + \sqrt{6})$$



$$x^2 - 2x - 3 > 0 \quad (3)$$

~~fact~~ fakt $\checkmark$ - L_-

$$x^2 - 2x - 3 = (x - 3)(x + 1)$$



so (3) has Lösung

$$L_3 = (-\infty, -1) \cup (3, \infty)$$

(1) has Lösung mit

$$L_1 = L_2 \cap L_3$$

$$\underline{\underline{= (1 - \sqrt{6}, -1) \cup (3, 1 + \sqrt{6})}}$$