Lecture 3

1. Functions

A function f on a set D onto a set S is a rule that assigns a unique element $f(x)$ in S to each element x in D.

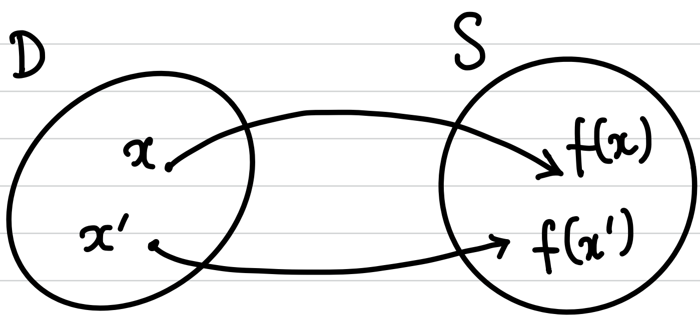
For example, in the parabola $y = x^2$ y is a function of x, we write

$$y = f(x) = x^2.$$

Ways to represent a function

$y = x^2$, $f(x) = x^2$ or $x \mapsto x^2$

(x goes to x^2)



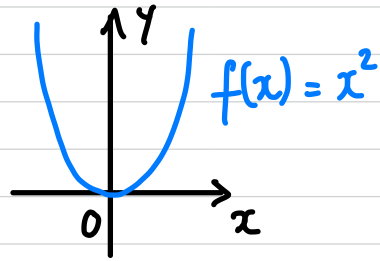
$D(f)$ domain of the function f:
set of all possible input.

$R(f)$ range of the function f:
set of all possible output.

Ex: $f(x) = x^2$

$D(f) = \mathbb{R}$

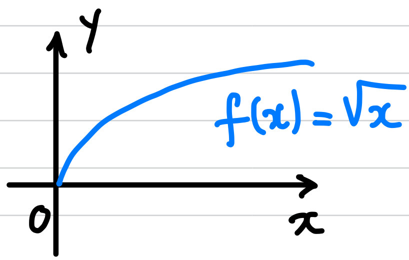
$R(f) = [0, +\infty)$



$f(x) = \sqrt{x}$

$D(f) = [0, +\infty)$

$R(f) = [0, +\infty)$



$f(x) = \sqrt{1-x^2}$

We need $1 - x^2 \geq 0 \iff x^2 \leq 1$

$|x| \leq 1$

$-1 \leq x \leq 1$

$$D(f) = \{x \in \mathbb{R} : -1 \leq x \leq 1\} = [-1, 1]$$

$$R(f) = [0, 1]$$

$$f(x) = \frac{1}{1-x}$$

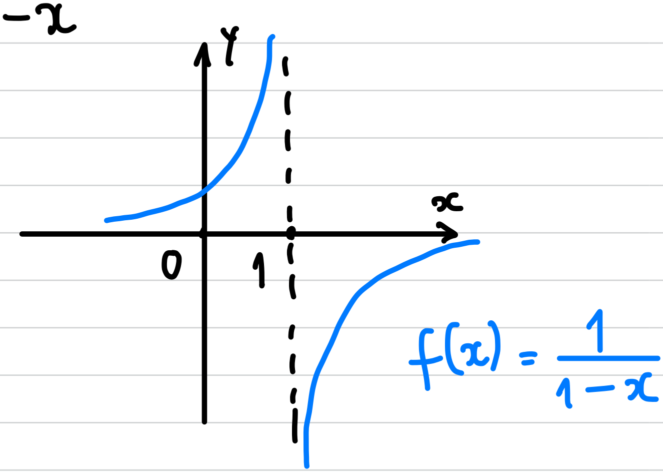
$$D(f) = \{x \in \mathbb{R} : x \neq 1\}$$

$$= (-\infty, 1) \cup (1, +\infty)$$

$$= \mathbb{R} \setminus \{1\}$$

$R(f) = \mathbb{R} \setminus \{0\}$ This because

$\dfrac{1}{1-x} = 0$ has no solution.



Domain convention

When a function f is defined without specifying its domain, we assume that the domain consists of all $x \in \mathbb{R}$ for which the $f(x)$ is a real number.

Remark: The square root function

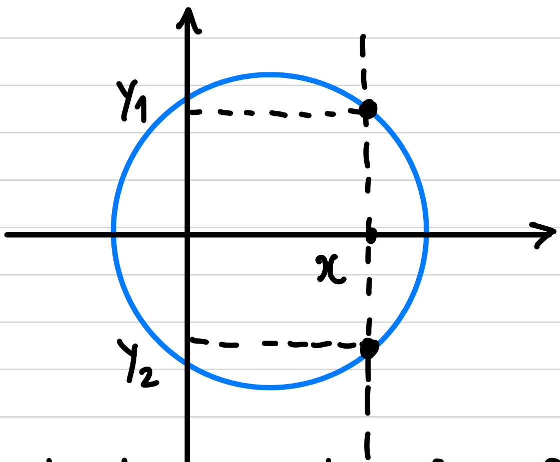
$f(x) = \sqrt{x}$ where $\sqrt{x}$ denotes the non-negative number whose square is x.

$$D(f) = [0, +\infty) \text{ and } R(f) = [0, +\infty)$$

2. Graph of a function: is the graph of equation $y = f(x)$

Not every curve is the graph of a function.

The graph of a function satisfies that no vertical line can intersect the graph at more than one point.



Circle is not the graph of a function.

3. Even and odd functions

Definition

The function f is even if

$$f(-x) = f(x) \text{ for all } x, -x \in D(f)$$

The function f is odd if

$f(-x) = -f(x)$ for all $x, -x \in D(f)$

Properties

Even function is symmetric about the y-axis.

Odd function is symmetric about the origin.

Examples

$f(x) = x^2 + 2$ is even

$$f(-x) = (-x)^2 + 2 = x^2 + 2 = f(x)$$

$f(x) = x^3$ is odd

$$f(-x) = (-x)^3 = (-1)^3 x^3 = -x^3 = -f(x)$$

$f(x) = (x+2)^2$ is neither even nor odd

$f(-x) = (-x+2)^2$ not $f(x)$ nor $-f(x)$