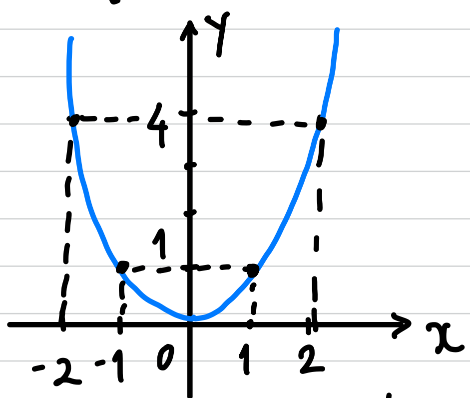
6. Graph: The graph of equation is the set of points where it coordinates satisfy the equation.

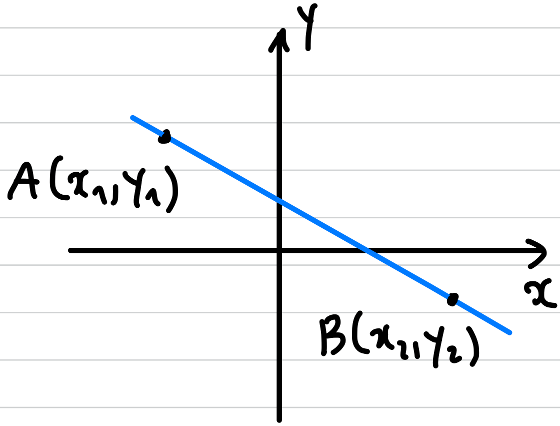


Graph of the equation $y = x^2$

Straight lines: 2 points determine one straight line.

The slope of a line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \left(\text{or } \frac{\Delta y}{\Delta x}\right)$$

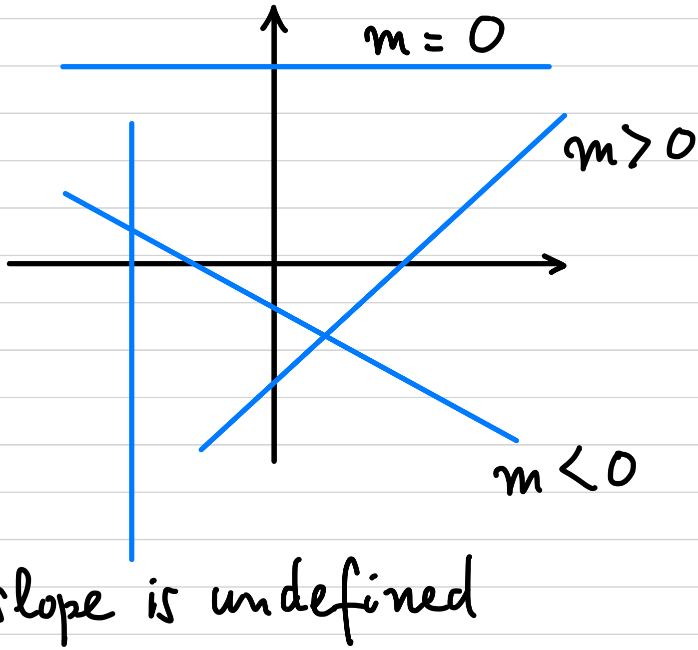


If the line raises from left to right then $m > 0$.

If the line falls from left to right then $m < 0$.

The slope of the horizontal line (when $y_2 = y_1$) is $m = 0$.

The slope of the vertical line (when $x_2 = x_1$) is undefined



7. Equations of lines

The equation of the line passes through the point (x_1, y_1) and has slope m is given by

$$y = m.(x - x_1) + y_1$$

(point-slope equation)

General linear equation in x and y

$$Ax + By = C$$

where A and B are not both zero.

See example 12 / page 16 textbook.

Lecture 2

Quadratic equations:

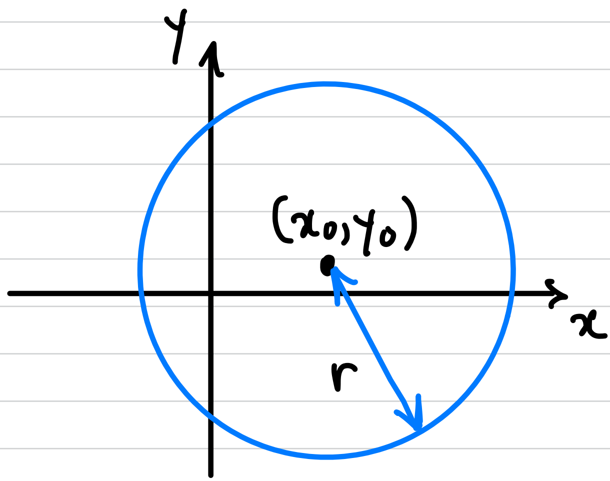
$$Ax^2 + By^2 + Cxy + Dx + Ey = F$$

1. Circles

The circle having center (x_0, y_0) and radius r has equation

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

The set of all points (x, y) that are at distance r from the point (x_0, y_0)



<u>Remark</u>: A quadratic form

$$x^2 + y^2 + 2ax + 2by = c$$

must represent a circle.

We write

$$(x+a)^2 + (y+b)^2 = c + a^2 + b^2$$

If $c + a^2 + b^2 > 0$ then the circle has center $(-a, -b)$ and radius

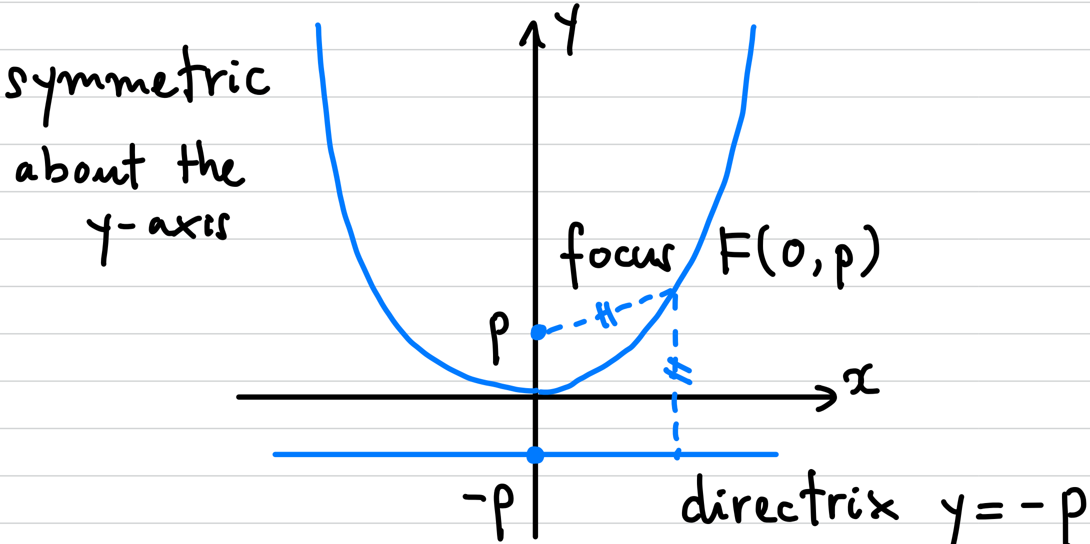
$$r = \sqrt{c + a^2 + b^2}.$$

If $c + a^2 + b^2 < 0$ then there is no graph.

If $c + a^2 + b^2 = 0$ then there is only one point $(x, y) = (-a, -b)$.

2. Parabolas

A parabola is the set of all points in the plane equidistant from a given line L and a given point F not on the line.

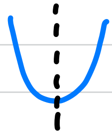


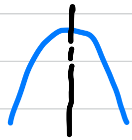
Parabola with focus F(0,p) and directrix line $y = -p$ $(p \neq 0)$

$$y = \frac{x^2}{4p}$$

$p > 0$ opens upward

$p < 0$ opens downward

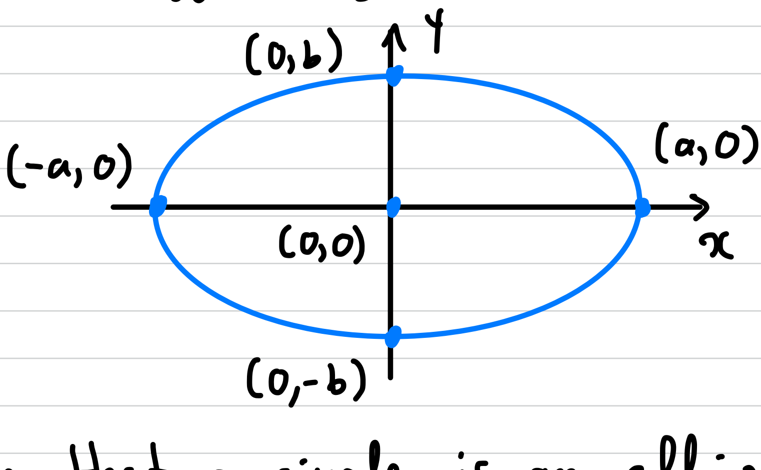




3. Ellipses

Ellipse with center $(0,0)$ and passes through 4 points $(a,0)$; $(0,b)$ $(0,-b)$ and $(-a,0)$ has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



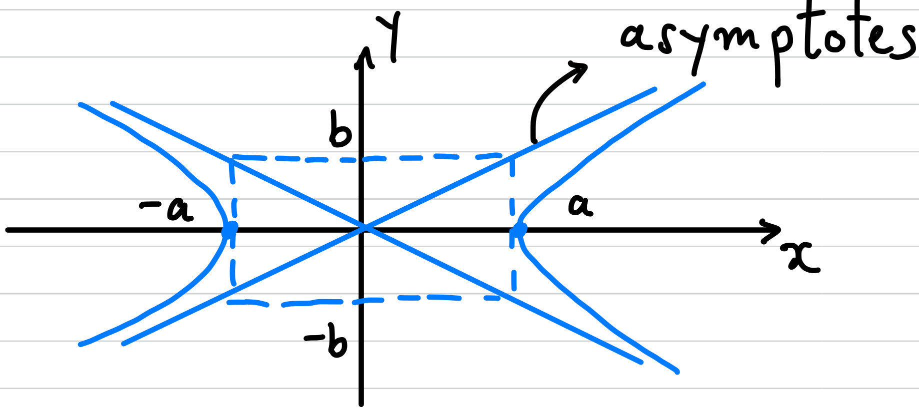
Note that a circle is an ellipse

$$x^2 + y^2 = r^2 \iff \frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

4. Hyperbolas

The equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

represent the hyperbola with center $(0,0)$ and passes through two points $(-a, 0)$ and $(0, a)$



5. Transformation of graphs

a. Shifts (translations)

For $c > 0$, if the graph of $y=f(x)$ is

translated	new equation
upward c units	$y = f(x) + c$
downward c units	$y = f(x) - c$
left c units	$y = f(x + c)$
right c units	$y = f(x - c)$

b. Stretches and compressions

For $c > 1$, if the graph of $y = f(x)$ is

expanded vertically by a factor of c	$y = c \cdot f(x)$
compressed vertically by a factor of c	$y = \frac{1}{c} \cdot f(x)$

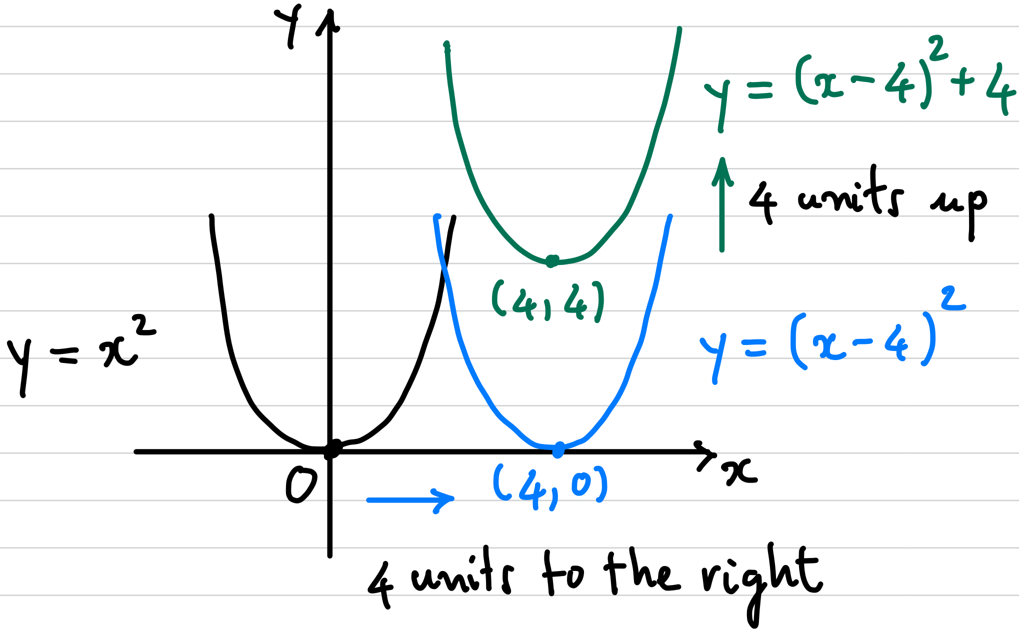
expanded horizontally by a factor of c	$y=f(cx)$
compressed horizontally by a factor of c	$y=f(\frac{x}{c})$

c. Reflection

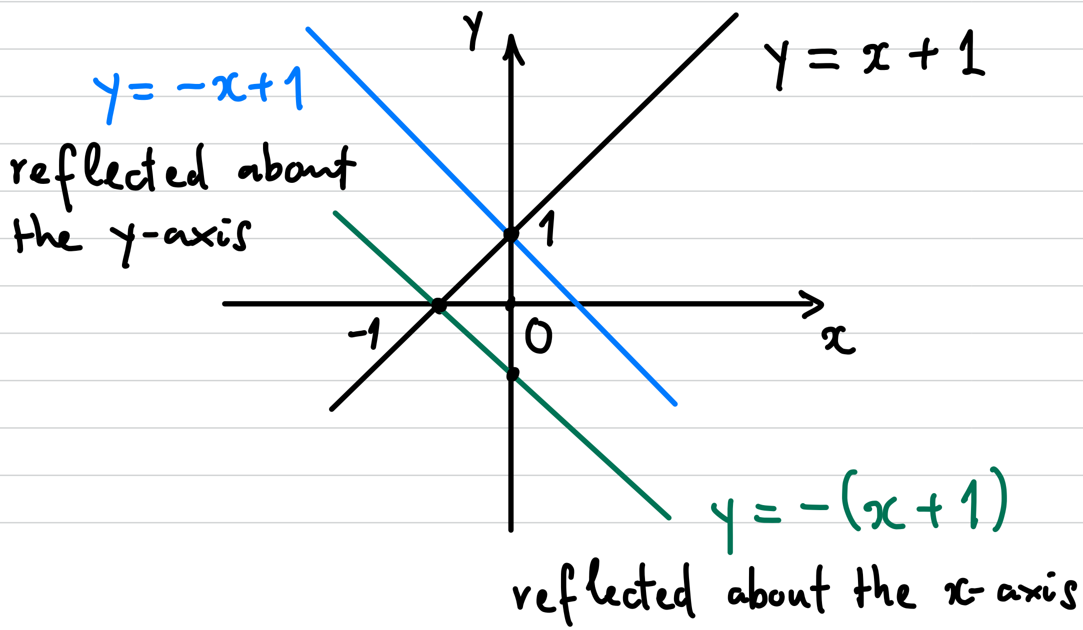
If the graph of $y=f(x)$ is

reflected about the x-axis	$y=-f(x)$
reflected about the y-axis	$y=f(-x)$
reflected through the origin O	$y=-f(-x)$

Examples



y
x
O
$y = x^2$
$y = (x-4)^2 + 4$
4 units up
(4, 4)
$y = (x-4)^2$
(4, 0)
4 units to the right



y
x
$y = x + 1$
$y = -x+1$
reflected about the y-axis
1
-1
0
$y = -(x+1)$
reflected about the x-axis