Constants and formulas
Space Physics EF2240

PHYSICAL CONSTANTS

c	speed of light	3×10^{8} ms^{-1}
e	elementary charge	1.6×10^{-19} C
m_e	electron mass	0.91×10^{-30} kg
m_p	proton mass	1.67×10^{-27} kg
h	Planck's constant	6.625×10^{-34} Js
k_B	Boltzmann's constant	1.38×10^{-23} JK^{-1}
μ_0	permeability of free space	$4\pi \times 10^{-7}$ Hm^{-1}
ε_0	permittivity of free space	8.854×10^{-12} Fm^{-1}
R_E	Earth's radius	6378 km
a	Earth's magnetic dipole moment	8×10^{22} Am^2
σ_{SB}	Stefan-Boltzmann's constant	5.67×10^{-8} W·m^{-2}·K^{-4}

CONVERSIONS

1 eV	electron volt	1.6×10^{-19} J
1 AU	astronomical unit	1.496×10^{11} m
1 γ	Gamma	10^{-9} T
1 G	Gauss	10^{-4} T
1 LY	light year	9.461×10^{15} m

PLASMA PHYSICS

ω_p	plasma frequency	$\sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$
ω_c	cyclotron-/gyro frequency	$\frac{eB}{m_e}$
r_L	gyro/Larmor radius	$\frac{mv_\perp}{eB}$
μ	magnetic moment	$\frac{mv_\perp^2}{2B}$
λ_D	Debye length	$\sqrt{\frac{\varepsilon_0 k_B T_e}{n_e e^2}}$
v_A	Alfvén velocity	$\frac{B}{\sqrt{\mu_0 \rho}}$
p_B	magnetic pressure/energy density	$\frac{B^2}{2\mu_0}$
p_d	dynamic pressure	ρv^2
p	thermal pressure	$n k_B T$
H	Atmospheric scale height	$\frac{k_B T}{mg}$
$\mathbf{v}_D$	drift velocity	$\frac{\mathbf{F} \times \mathbf{B}}{qB^2}$
R_m	magnetic Reynolds number	$\mu_0 \sigma v_c l_c$
n	index of refraction for plasma	$\sqrt{1 - \frac{f_{pe}^2}{f^2}}$
α_{lc}	loss cone angle	$\arcsin \sqrt{B / B_{\max}}$

PLASMA ELECTRODYNAMICS

$j_{//}$	parallel current density	$\sigma_{//} E_{//}$
j_P	Pedersen current density	$\sigma_P E_\perp$
j_H	Hall current density	$\sigma_H E_\perp$
$\sigma_{//}$	parallel conductivity	$\sigma_{//} = \sigma_e + \sigma_i$
σ_P	Pedersen conductivity	$\sigma_e \frac{1}{1+\omega_{ge}^2 \tau_e^2} + \sigma_i \frac{1}{1+\omega_{gi}^2 \tau_i^2}$
σ_H	Hall conductivity	$\sigma_e \frac{\omega_{ge}\tau_e}{1+\omega_{ge}^2 \tau_e^2} - \sigma_i \frac{\omega_{gi}\tau_i}{1+\omega_{gi}^2 \tau_i^2}$
σ_e	electron conductivity	$e^2 n \tau_e / m_e$
σ_i	ion conductivity	$e^2 n \tau_i / m_i$

Current density in magnetized plasma

$$\mathbf{j} = \sigma_{\|}\mathbf{E}_{\|} + \sigma_P \mathbf{E}_{\perp} + \sigma_H \frac{\mathbf{B}\times\mathbf{E}_{\perp}}{B}$$

Ampére's law for infinite current sheet

$$j_z = -\frac{1}{\mu_0}\frac{\partial B_x}{\partial y}$$

RELATIVISTIC FIELD TRANSFORMATIONS

Electric field **($\perp$u)** $$\mathbf{E}' = \frac{\mathbf{E}+\mathbf{u}\times\mathbf{B}}{\sqrt{1-u^2/c^2}}$$

Magnetic field **($\perp$u)** $$\mathbf{B}' = \frac{\mathbf{B}-\left(\mathbf{u}/c^2\right)\times\mathbf{E}}{\sqrt{1-u^2/c^2}}$$

The components parallel to u remain unchanged.

RELATIVISTIC MECHANICS

Gamma factor $$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Relativistic momentum

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m \mathbf{v}$$

Relativistic energy $$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma mc^2$$

Rest energy $$E = mc^2$$

Relation between energy and momentum

$$E^2 = p^2c^2 + m^2c^4$$

OTHER

Dipole magnetic field

$$B_r = B_p\left(\frac{R_E}{r}\right)^3 \cos\theta$$

$$B_\theta = \frac{B_p}{2}\left(\frac{R_E}{r}\right)^3 \sin\theta$$

where B_p is the magnetic field strength at the poles

Dipole magnetic field, alternative formulation

$$B_r = \frac{\mu_0 a}{2\pi}\frac{1}{r^3}\cos\theta$$

$$B_\theta = \frac{\mu_0 a}{2\pi}\frac{1}{2}\frac{1}{r^3}\sin\theta$$

where a is the magnetic dipole moment

Parker spiral $$\frac{B_\theta}{B_r} = \tan\psi = \frac{\omega_{sun} r}{u_{sw}}$$

Energy for light quantum $$E = hf$$

Wien's displacement law

$$\lambda = \frac{2.90\times10^{-3}\ \mathrm{m\cdot K}}{T}$$

Stefan-Boltzmann's law $$J = \sigma_{SB} T^4$$

Dispersion relation for electromagnetic waves in vacuum $$c = f\lambda$$