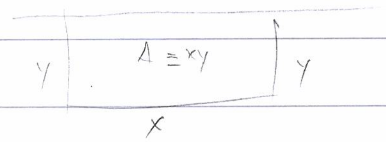
(67) Types of problems dealing with derivatives

Extreme Value problems

These problems deal with relating a problem to finding the maximum or minimum of a function

Ex! A rectangular animal enclosure is to be constructed having one side along an existing long wall and the other three sides fenced. If 100m of fence are available what is the largest possible area for the enclosure?

Solution



Maximize $A = xy$

such that $x + 2y = 100$

i.e. $A = (100-2y)y = 100y - 2y^2$

$A' = 100 - 5y \Rightarrow y = 20,\ x = 60$

$A'' = -4y < 0$ so max

Tips to solve Extreme value problems

1. Extreme value problem are commonly graphized, so first draw a diagram
2. Fully label your diagram with all information given
3. Identify what you need to maximize or minimize
4. Find a relation relating what you want with the other parameters.
5. Typically have more than one parameter, so find another relation relating the parameters.
6. Substitute so that what you want is given by just one parameter

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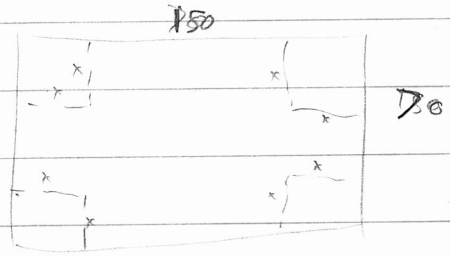
(7) Differentiate with respect to this isolated parameter and find critical values

(8) Check which critical values give maxima or minima and which make physical sense

Examples p. 267 #18



$$V = (150-2x)(70-2x)\cdot x$$
$$= 10500x - 440x^2 + 4x^3$$
$$V' = 10500 - 880x + 12x^2$$
$$= 4(x-15)(3x-175)$$

$x=15 \Rightarrow V = 72\,000$

$x = \frac{175}{3}$ does not make sense since $\frac{2\cdot 175}{3} \geq 70$

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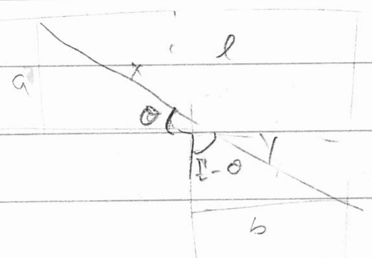

$$l = x + y$$

$\sin\theta = \frac{a}{x}$ $\qquad$ $\sin\left(\frac{\pi}{2}-\theta\right) = \frac{b}{y}$, $\cos\theta$

$\Rightarrow x = \frac{a}{\sin\theta}$ $\qquad$ $y = \frac{b}{\cos\theta}$

$$l = \frac{a}{\sin\theta} + \frac{b}{\cos\theta}$$

$$l' = \frac{-a\cos\theta}{(\sin\theta)^2} + \frac{b\sin\theta}{(\cos\theta)^2}$$
$$= \frac{-a(\cos\theta)^3 + b(\sin\theta)^3}{(\sin\theta)^2(\cos\theta)^2}$$

$\left(\frac{\sin\theta}{\cos\theta}\right)^3 = \frac{b}{a}$ $\qquad$ $\theta = \tan^{-1}\left(\sqrt[3]{\frac{b}{a}}\right)$ $\qquad$ $\sin(\tan^{-1}) = \frac{x}{\sqrt{x^2+1}}$

$\cos(\tan^{-1}) = \frac{1}{\sqrt{x^2+1}}$

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Types of problems using derivatives

Related rates:

When several quantities vary with time we can differentiat them with respect to time. Then, if these quantities satisfy some relationships we can determine how fast one is changing by how fast the others are changing.

Ex: How fast is the area of a rectangle changing when one side is 10 cm and is increasing at a rate of 2cm/s and the other side is 8cm and is decreasing at a rate of 3cm/s.

Ans: $A = xy \rightarrow \frac{d}{dt} A = \frac{d}{dt} x \cdot y + x \cdot \frac{d}{dt} y$

$= 2 \cdot 8 - 10 \cdot 3 = 16 - 30 = -14$

Tips to solve related rate questions.

1. Related rates questions are typically graphical, so the first step is to draw a diagram
2. Fully label your drawing with all information given
3. Identify what you need to determine
4. Find a relation that relates what you are looking for to what you know
5. Differentiate and solve.

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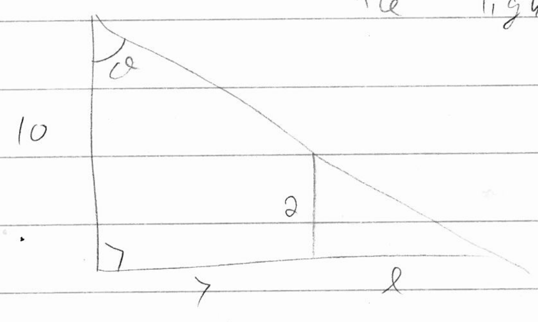
Ex: There is a spotlight at the top of a pole 10 m high. It takes one minute to go from pointing straight down to pointing straight forward. If a 2 m tall man is standing 7 m away from the pole, how fast is his shadow growing, when the light is halfway through its rotation?



$$\frac{d\theta}{dt} = \frac{\pi}{2} \text{ rad/min}$$

$$\tan\theta = \frac{7+\ell}{10}$$

$$\Rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{d\ell}{dt}$$

$$\frac{d\ell}{dt} = 2 \cdot \frac{\pi}{2} = \pi$$

Ex: #26 p.221 #35

Indeterminate forms & l'Hôpital's rule

there are many forms of limits that we still don't have a good grasp on solving because they approach something which doesn't make sense i.e. an indeterminate form