Department of Mathematics

SF1624/SF1684
Algebra and Geometry
Year 2019/2020

Problems for Seminar 3

Check the canvas page of the course for information on how seminars are run and what you are expected to do before and during the seminars.

The seminar starts with a test. The problem will be about computing a determinant by cofactor expansion and/or Gauss elimination.

In the seminar, the following problems will be discussed.

Problem 1. Let $Ax = b$ be a system of linear equations, where A is an $m \times n$-matrix (that is, m equations in n variables), and $Ax = 0$ the associated homogeneous system of equations. Explain why each one of the following claims is true or false:

- If $Ax = 0$ has a nonzero solution then $Ax = b$ also has a solution, irrespective of m and n.
- If $Ax = b$ has a solution then $Ax = 0$ has a nonzero solution, irrespective of m and n.
- If $m < n$ then $Ax = b$ cannot have a unique solution.
- If $m > n$ then $Ax = b$ cannot have a unique solution.
- If $Ax = b$ has a unique solutions then $x = 0$ is the only solution for $Ax = 0$.

A simple counterexample is the best explanations for why a claim is false!

Problem 2. Matrisen

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

är ett speciallfall av en typ av matriser som ofta förekommer i olika tillämpningar, exempelvis i samband med diskretisering av differentialekvationer för numerisk lösning. Använd rad- eller kolonnoperationer för att beräkna determinanten av matrisen A.

Problem 3. The four vectors

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 0 \end{bmatrix} \quad \text{och} \quad \vec{u}_4 = \begin{bmatrix} 3 \\ 5 \\ 3 \\ 0 \end{bmatrix}$$

in $\mathbb{R}^4$ are linearly dependent. Write on of them as a linear combimation of the others.

Problem 4. *Tetrahedron* is a three dimensional object with four vertices whose sides are triangles. The four points $O = (0, 0, 0)$, $A = (1, 2, -3)$, $B = (3, 1, 0)$ and $C = (0, 2, 1)$ are the vertices of the tetrahedron. Volume of tetrahedron can be computed as one sixth of the absolute value of the tripple product, $(\vec{u} \times \vec{v}) \cdot \vec{w}$, of the three vectors which go from one vertex of the tetrahedron to the three others.

(a) Compute the cross product of the two vectors $\vec{u} = \overrightarrow{OA}$ and $\vec{v} = \overrightarrow{OB}$

(b) Compute the volume of the tetrahedron which has vertices at points O, A, B och C.

MISCELLANEOUS

Here are some other topics that are important and interesting to discuss.

- What is the definition of linear independence? What are equivalent ways of expressing this?
- Which method is best for computing a determinant?
- Is is possible to define the cross product for vectors of length other than 3 in a reasonable way?