Problems for Seminar 2

Check the canvas page of the course for information on how seminars are run and what you are expected to do before and during the seminars.

The seminar starts with a test. The problem will be about solving a system of linear equations or converting a matrix to row echelon form.

In the seminar, the following problems will be discussed.

Problem 1. For each number a, vi have a system of equations in three variables x, y, och z, given by

$$(\star) \quad \begin{cases} (a-3)y &=& 1, \\ 2x - ax + ay - 3y + 2z - az &=& 1, \\ (4-2a)x + (2a-6)y + 5z - 2az &=& 3. \end{cases}$$

Show the the system of equations $(\star)$ has a unique solution if and only if $a \neq 2$ and $a \neq 3$. Solve the system $(\star)$ for $a = 2$ using row operations on the matrix for the system.

Problem 2. Consider the system of linear equations

$$(1) \quad \begin{cases} 17x &-& 13y &+& 2z &-& 7w &=& 5, \\ 13x &+& 6y &-& z &+& 11w &=& 3. \end{cases}$$

(a) Determine a solution for the system when $x = 0$ and $w = 1$.

(b) Explain why the system (1) has infinitely many solutions.

(c) Does there exist a solution to the system when $y = -2x$ and $w = -3x$?

Problem 3. Let

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}.$$

(a) A matrix S is said to be a *square root* of a matrix M if $SS = M$. Find two square roots of A.

(b) How many different square roots of B can you find?

(c) Do you think that every matrix has a square root? Explain your reasoning.

from Anton-Busby, Ex. 3.1.D6

MISCELLANEOUS

Here are some other topics that are important and interesting to discuss.

- Comments on Seminar 1: Anything that was unclear (theory, grading)?
- How does Gauss-Jordan elimination affect the solution set? What is the purpose of Gauss-Jordan elimination?
- What is the connection between solutions of inhomogeneous systems of equations and solutions of the associated homogeneous system?
- Pivots cannot be zero, otherwise one has to perform a change of rows. What happens if a pivot is not zero but very small? Is it okay to change rows even if the pivot is nonzero? Can this be advantageous?