Department of Mathematics

SF1624/SF1684
Algebra and Geometry
Year 2019/2020

Problems for Seminar 1

Check the canvas page of the course for information on how seminars are run and what you are expected to do before and during the seminars.

The seminar starts with a test. The problem will be about finding an equation for a line or a plane in $\mathbb{R}^3$ with certain properties.

In the seminar, the following problems will be discussed.

Problem 1. The points P, Q, and R have coordinates

$$P = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad Q = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad R = \begin{bmatrix} 4 \\ 6 \\ -1 \end{bmatrix},$$

and the plane Π is given by the equation

$$2x - y + 2z = 3.$$

(a) Compute a parameter form for the line L through P and Q.
(b) Does the line L contain the point R?
(c) Determine the intersection of Π and L.
(d) Compute an equation for the plane which is orthogonal to L and contains P.

Problem 2. For each number t, we are given a triangle T with vertices

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} t \\ t \\ 1 \end{bmatrix}.$$

(a) For which value of t is T right-angled at A?
(b) Compute the other two angles for this value of t (use a calculator).
(c) Find a value for t such that the point C is closest to A.

Problem 3. Find an equation for the plane consisting of all points with equal distance to the point $A = (-1, 1, 2)$ and to the point $B = (1, 5, -4)$. (*Hint*: The mid point of the line segment between A and B lies in the plane.)

MISCELLANEOUS

Here are some other topics that are important and interesting to discuss.

- How many equations does one need to define an m-dimensional subspace of $\mathbb{R}^n$? How many free variables are needed in a parametric form?
- What is the expected intersection between two planes in $\mathbb{R}^3$, in $\mathbb{R}^4$, and in $\mathbb{R}^5$?
- What might the angle between a line and a plane or between two planes in $\mathbf{R}^3$ mean? How can one compute this angle?