Tutorial 1: FEM for Engineering Applications (SE1025)

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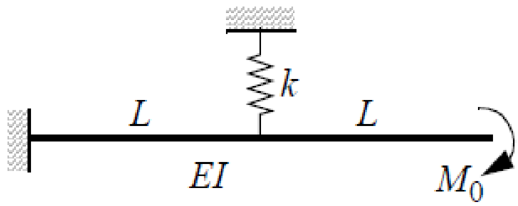
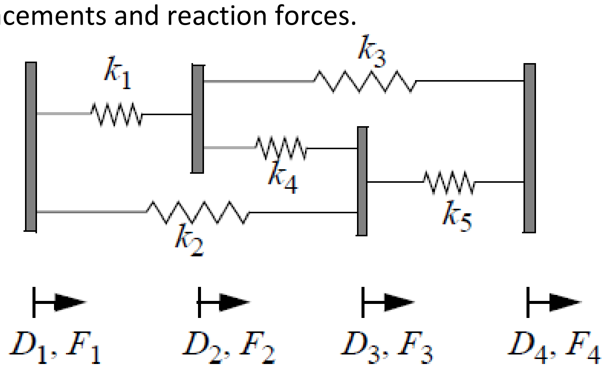
Problems:

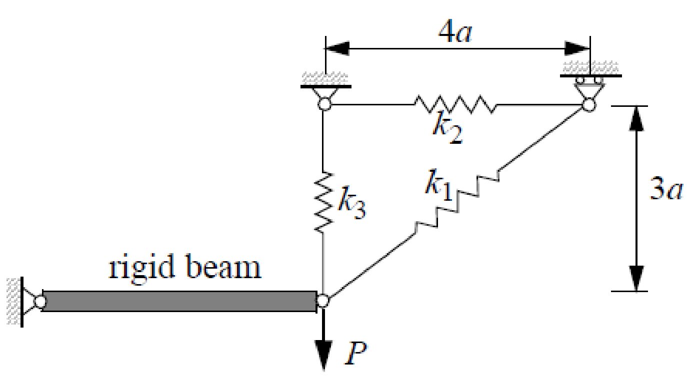
1.6 A beam of length *2L* and with bending stiffness *EI* is subjected to a bending moment applied at its right end. The left end of the beam is clamped, and the mid-point is attached to a vertical spring with spring constant $k = 6EI/L^3$. Evaluate the rotation of the right end of the beam by use of an energy method.



2.1 A system of five springs are connected as shown in the figure. All the spring constants are in the current application equal to *k*. Furthermore, *D1 = D4 = 0, F3 = 0* and *F2 = P*. Determine the displacements and reaction forces.



2.2 Three springs are connected according to the figure, also showing the applied external force *P*. The spring constants are: $k_1 = 5k$, $k_2 = k$ and $k_3 = 2k$. Determine the displacements at the points where the springs are connected and evaluate all the reaction forces.



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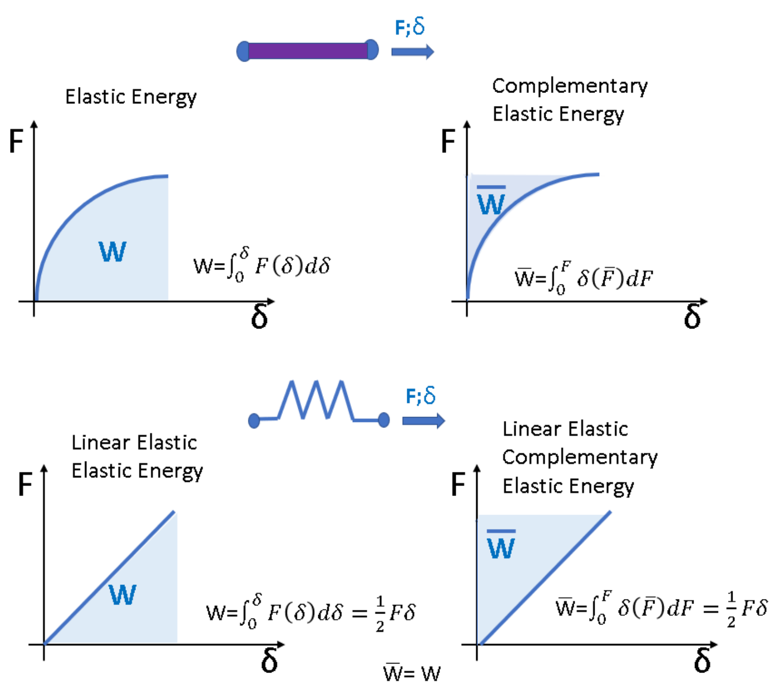
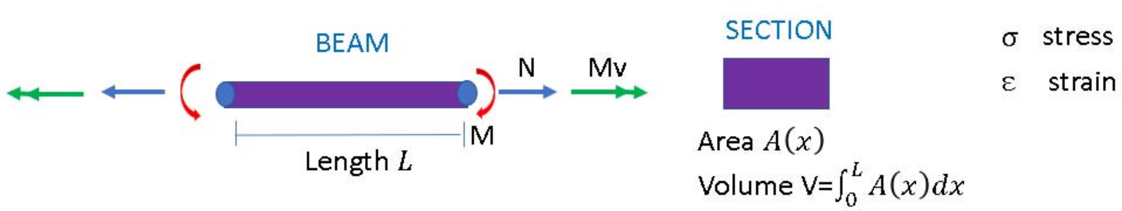
Summary from the classes (1):

Elastic energy and Complementary elastic energy





Linear Elastic Energy

$$W=\int_0^\delta F(\delta)d\delta = \frac{1}{2}F\delta$$

$$W=\int_V \int_0^\varepsilon \sigma(\varepsilon)\,d\varepsilon\,dV = \int_V \int_0^\varepsilon E\varepsilon\,d\varepsilon\,dV = \int_V \frac{1}{2}E\varepsilon^2\,dV = \int_V \frac{1}{2}\sigma\varepsilon\,dV = \int_0^L \frac{1}{2}\sigma(x)\varepsilon(x)\,A(x)\,dx$$

Generally A(x) constant

$$A(x) = A$$

Linear Complementary Elastic Energy

$$\overline{W}=\int_0^F \delta(\bar{F})dF = \frac{1}{2}F\delta$$

$$\overline{W}=\int_V \int_0^\sigma \varepsilon(\sigma)\,d\sigma\,dV = \int_V \int_0^\sigma \frac{\sigma}{E}\,d\sigma dV = \int_V \frac{1}{2}\frac{\sigma^2}{E}dV = \int_V \frac{1}{2}\sigma\varepsilon\,dV = \int_0^L \frac{1}{2}\sigma(x)\varepsilon(x)\,A(x)\,dx$$

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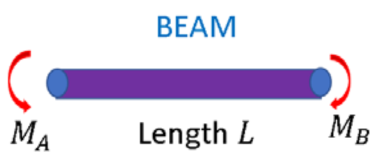
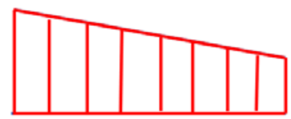
$$\overline{W}=\int_0^L \underbrace{\boxed{\frac{N(x)^2}{2EA(x)}}}_{\text{Normal force}} + \underbrace{\boxed{\frac{M(x)^2}{2EI(x)}}}_{\text{Bending Moment}} + \underbrace{\boxed{\frac{M_v(x)^2}{2GK(x)}}}_{\text{Torque}} + \ldots dx$$





$$M(x) = M_A + \frac{M_B - M_A}{L} x$$

$$\overline{W}=\int_0^L \frac{M(x)^2}{2EI(x)} dx = \frac{L}{6EI}(M_A^2 + M_A M_B + M_B^2)$$

A constant

I constant

Castigliano 2nd Theorem

$$q_k = \frac{\partial \overline{W}}{\partial Q_k}$$

q_k Unknown Displacement/Rotation

Q_k Known Force/Moment

Principle of Least Work

$$\frac{\partial \overline{W}}{\partial R_k} = 0$$

R_k Reaction Force

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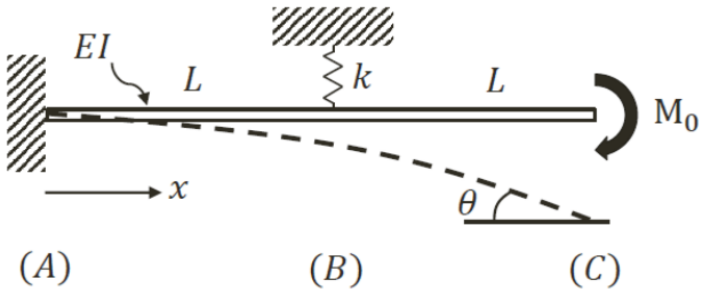
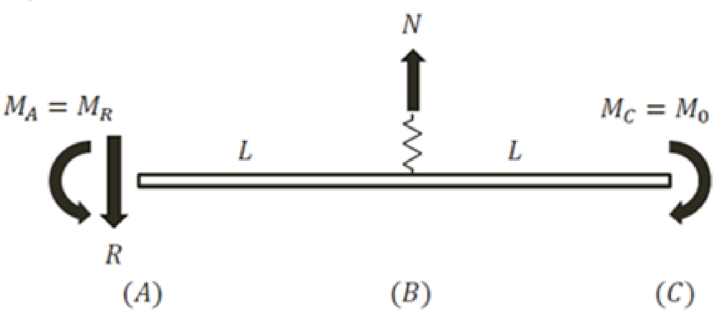
Problem 1.6



Castigliano 2nd Theorem

$$\vartheta = \frac{\partial \overline{W}_{TOT}}{\partial M_0} \Rightarrow \overline{W}_{TOT} = \overline{W}_{Beam} + \overline{W}_{Spring}$$

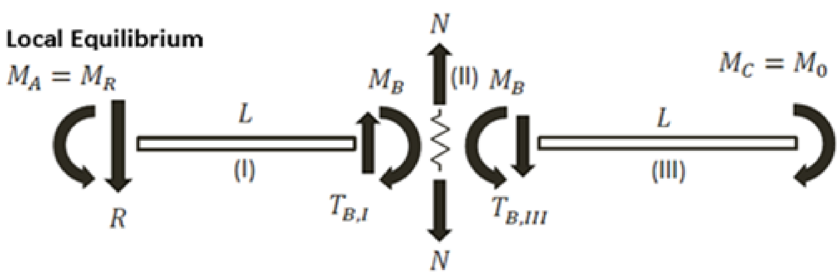
Equilibrium of the structure



$\uparrow: -R + N = 0$
$\Leftrightarrow R = N$

$\circlearrowright_A: -M_R - N \cdot L + M_0 = 0$

Local Equilibrium



$\uparrow_I: -R + T_{B,I} = 0$
$\Leftrightarrow T_{B,I} = R$

$\circlearrowright_{B,III}: -M_B + M_0 = 0$
$\Leftrightarrow M_B = M_0$

From the equilibrium

$$\begin{cases} M_R = M_0 - NL \\ M_A = M_R \\ M_B = M_0 \\ R = N \end{cases}$$

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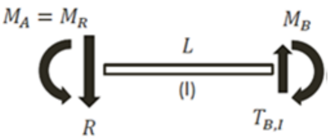
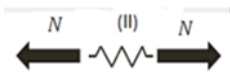
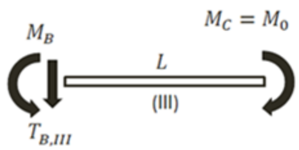


$$\bar{W}_{(I)}=\int_0^L \frac{M(x)^2}{2EI(x)}dx = \frac{L}{6EI}(M_A^2 + M_A M_B + M_B^2) =$$
$$= \frac{L}{6EI}(M_R^2 + M_R M_0 + M_0^2) = \frac{L}{6EI}[(M_0 - NL)^2 + (M_0 - NL)M_0 + M_0^2] =$$
$$= \frac{L}{6EI}[M_0^2 + N^2L^2 - 2M_0NL + M_0^2 - M_0NL + M_0^2] = \frac{L}{6EI}[3M_0^2 + N^2L^2 - 3M_0NL]$$



$$\bar{W}_{(II)}=\frac{1}{2}N\delta = \frac{1}{2}N\left(\frac{N}{k}\right)=\frac{1}{2}\frac{N^2L^3}{6EI}=\frac{1}{12}\frac{N^2L^3}{EI}=\frac{L}{6EI}\left(\frac{N^2L^2}{2}\right)$$



$$\bar{W}_{(III)}=\int_0^L \frac{M(x)^2}{2EI(x)}dx = \frac{L}{6EI}(M_B^2 + M_B M_C + M_C^2) =$$
$$= \frac{L}{6EI}(M_0^2 + M_0^2 + M_0^2) = \frac{L}{6EI}(3M_0^2)$$

$$\bar{W}_{TOT} = \bar{W}_{(I)} + \bar{W}_{(II)} + \bar{W}_{(III)} = \frac{L}{6EI}\left(6M_0^2 + \frac{3}{2}N^2L^2 - 3M_0NL\right)$$

Principle of Least Work

$$\frac{\partial \bar{W}_{TOT}}{\partial N} = 0 \Rightarrow \frac{L}{6EI}(3N^2L^2 - 3M_0L) = 0 \Rightarrow N = \frac{M_0}{L}$$

So…

$$\bar{W}_{TOT} = \bar{W}_{(I)} + \bar{W}_{(II)} + \bar{W}_{(III)} = \frac{L}{6EI}\left(6M_0^2 + \frac{3}{2}M_0^2 - 3M_0^2\right) = \frac{3}{4}\frac{M_0^2L}{EI}$$

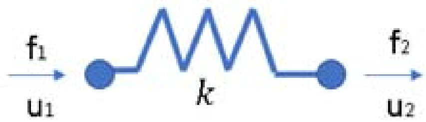
Castigliano 2nd Theorem

$$\boxed{\vartheta = \frac{\partial \bar{W}_{TOT}}{\partial M_0} = \frac{3}{2}\frac{M_0L}{EI}}$$

$$\frac{[Nm][m]}{\left[\frac{N}{m^2}\right][m^4]} = 1\ OK$$

Summary from the classes (2):

Matrix Formulated Structural Mechanics: 1D



$$W=\int_0^{\delta} F(\delta)d\delta = \frac{1}{2}F\delta = \frac{1}{2}k\delta^2 = \frac{1}{2}k(u_2-u_1)^2 = \frac{1}{2}k(u_2^2+u_1^2-2u_1u_2)$$

Castigliano 1st Therem

$$f_1 = \frac{\partial W}{\partial u_1} = ku_1 - ku_2$$

$$f_2 = \frac{\partial W}{\partial u_2} = -ku_1 + ku_2$$

$$\Rightarrow \quad \underbrace{\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}}_{\mathbf{K_e}} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{\mathbf{D_e}} = \underbrace{\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}}_{\mathbf{F_e}}$$

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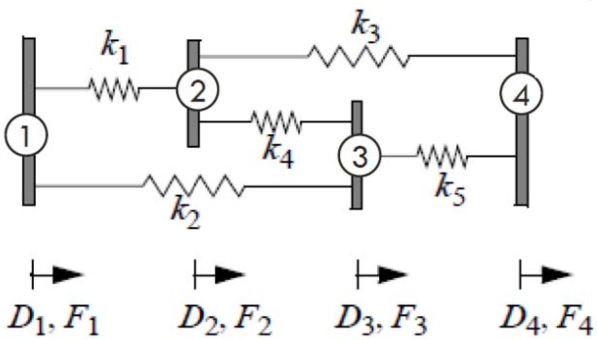
Problem 2.1



What we want:

Forces: $\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$ Displ: $\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$

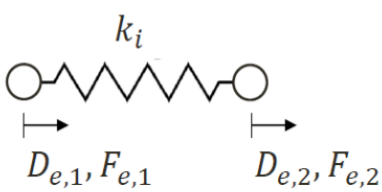
Data:

$$\begin{cases} D_1 = 0 \\ F_2 = P \\ F_3 = 0 \\ D_4 = 0 \end{cases}$$

Material Data

$$k = k_1, k_2, k_3, k_4, k_5$$

For a single element

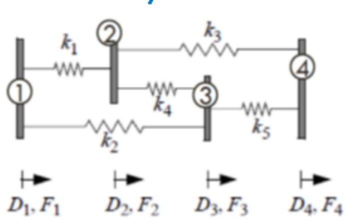


$$\mathbf{F_e} = \mathbf{K_e D_e}$$

$$\Rightarrow \begin{cases} F_{e,1} = k_i(+D_{e,1} - D_{e,2}) \\ F_{e,2} = k_i(-D_{e,1} + D_{e,2}) \end{cases} \Rightarrow \mathbf{K_{e,i}} = k_i \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$\begin{bmatrix} F_{e,1} \\ F_{e,2} \end{bmatrix} = k_i \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} D_{e,1} \\ D_{e,2} \end{bmatrix}$$

For the system



$$\mathbf{F} = \mathbf{KD}$$

$$\underset{\mathbf{F}}{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}} = \underset{\mathbf{K}}{\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}} \underset{\mathbf{D}}{\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}}$$

Assembling

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = k \begin{bmatrix} 1+1 & -1 & -1 & 0 \\ -1 & 1+1+1 & -1 & -1 \\ -1 & -1 & 1+1+1 & -1 \\ 0 & -1 & -1 & 1+1 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = k \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = k \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ D_2 \\ D_3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} P \\ 0 \end{bmatrix} = k \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} D_2 \\ D_3 \end{bmatrix}$$

Compute the displacements

$$\mathbf{F} = \mathbf{K}\mathbf{D} \qquad \mathbf{K}^{-1}\mathbf{F} = \mathbf{D}$$

$$\begin{bmatrix} D_2 \\ D_3 \end{bmatrix} = \frac{1}{k} \cdot \frac{1}{3 \cdot 3 - (-1)(-1)} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} P \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} D_2 \\ D_3 \end{bmatrix} = \frac{1}{8k} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} P \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{D_2} \\ \boldsymbol{D_3} \end{bmatrix} = \frac{\boldsymbol{P}}{\boldsymbol{8k}} \begin{bmatrix} \mathbf{3} \\ \mathbf{1} \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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Compute the forces

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = k \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} \frac{P}{8k} \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

$$F_1 = k\left[2 \cdot 0 - 1 \cdot \frac{3P}{8k} - 1\left(\frac{P}{8k}\right)\right]$$

$$\boldsymbol{F_1 = -\frac{P}{2}}$$

$$F_4 = k\left[-1 \cdot \frac{3P}{8k} - 1\left(\frac{P}{8k}\right) + 2 \cdot 0\right]$$

$$\boldsymbol{F_4 = -\frac{P}{2}}$$

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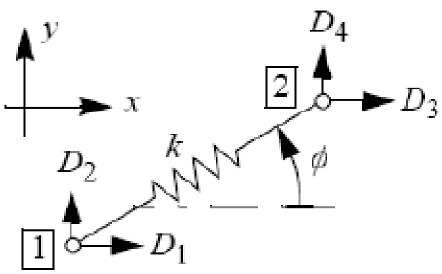
Matrix Formulated Structural Mechanics: 2D



$$\mathbf{K}_e = k\begin{bmatrix} \mathbf{a} & -\mathbf{a} \\ -\mathbf{a} & \mathbf{a} \end{bmatrix}$$

$$\mathbf{a} = \begin{bmatrix} c^2 & sc \\ sc & s^2 \end{bmatrix} \qquad \begin{aligned} c &= \cos\phi \\ s &= \sin\phi \end{aligned}$$

$$\mathbf{a} = \begin{bmatrix} l_{12}^2 & l_{12}m_{12} \\ l_{12}m_{12} & m_{12}^2 \end{bmatrix} \qquad \begin{aligned} l_{12} &= \cos\phi_x = (x_2 - x_1)/L \\ m_{12} &= \cos\phi_y = (y_2 - y_1)/L \\ L &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$

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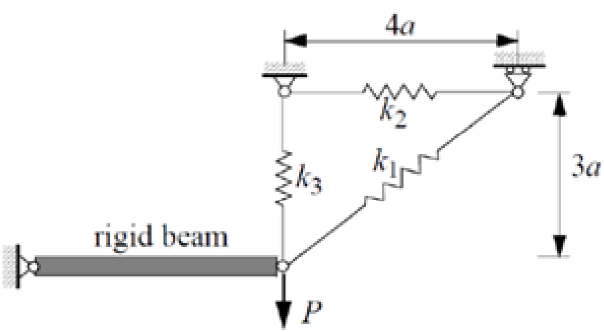
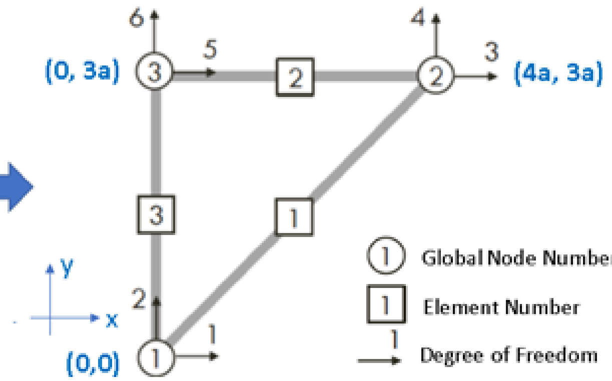
Problem 2.2





What we want:

Forces:

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{bmatrix}$$

Displ:

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} = \begin{bmatrix} D_{1x} \\ D_{1y} \\ D_{2x} \\ D_{2y} \\ D_{3x} \\ D_{3y} \end{bmatrix}$$

Data:

$$\begin{cases} D_1 = 0 \\ F_2 = -P \\ F_3 = 0 \\ D_4 = 0 \\ D_5 = 0 \\ D_6 = 0 \end{cases}$$

Material Data:

$$k_1 = 5k$$
$$k_2 = k$$
$$k_3 = 2k$$

Create the Stiffness matrix for each element

Elem.	k	x_1	x_2	y_1	y_2	L	l_{12}	m_{12}	a	$\mathbf{K_e}$
1	$5k$	0	$4a$	0	$3a$	$5a$	4/5	3/5	$\frac{1}{25}\begin{bmatrix} 16 & 12 \\ 12 & 9 \end{bmatrix}$	$\frac{k}{5}\begin{bmatrix} 16 & 12 & -16 & -12 \\ 12 & 9 & -12 & -9 \\ -16 & -12 & 16 & 12 \\ -12 & -9 & 12 & 9 \end{bmatrix}$
2	k	0	$4a$	$3a$	$3a$	$4a$	1	0	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$k\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
3	2k	0	0	0	$3a$	$3a$	0	1	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$2k\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$

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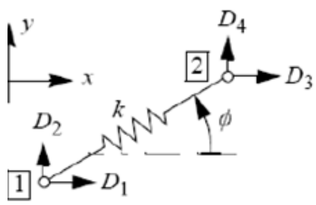

$$\mathbf{K}_e = k\begin{bmatrix} \mathbf{a} & -\mathbf{a} \\ -\mathbf{a} & \mathbf{a} \end{bmatrix} \qquad \mathbf{a} = \begin{bmatrix} l_{12}^2 & l_{12}m_{12} \\ l_{12}m_{12} & m_{12}^2 \end{bmatrix} \qquad \begin{aligned} l_{12} &= \cos\phi_x = (x_2 - x_1)/L \\ m_{12} &= \cos\phi_y = (y_2 - y_1)/L \end{aligned}$$

Assemble the Stiffness Matrix

Element	Node	Stiffness k_i	Element Stiffness Matrix $\mathbf{K}_e$
1	1, 2	$5k$	$k\begin{bmatrix} \frac{16}{5} & \frac{12}{5} & -\frac{16}{5} & -\frac{12}{5} \\ \frac{12}{5} & \frac{9}{5} & -\frac{12}{5} & -\frac{9}{5} \\ -\frac{16}{5} & -\frac{12}{5} & \frac{16}{5} & \frac{12}{5} \\ -\frac{12}{5} & -\frac{9}{5} & \frac{12}{5} & \frac{9}{5} \end{bmatrix}$
2	2, 3	k	$k\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
3	1, 3	2k	$k\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 \end{bmatrix}$

$\Rightarrow$

Row → Col. ↓	1 $1x$	2 $1y$	3 $2x$	4 $2y$	5 $3x$	6 $3y$
1 $1x$	$\frac{16}{5}$	$\frac{12}{5}$	$-\frac{16}{5}$	$-\frac{12}{5}$	0	0
2 $1y$	$\frac{12}{5}$	$\frac{9}{5}+2$	$-\frac{12}{5}$	$-\frac{9}{5}$	0	-2
3 $2x$	$-\frac{16}{5}$	$-\frac{12}{5}$	$\frac{16}{5}+1$	$\frac{12}{5}$	-1	0
4 $2y$	$-\frac{12}{5}$	$-\frac{9}{5}$	$\frac{12}{5}$	$\frac{9}{5}$	0	0
5 $3x$	0	0	-1	0	1	0
6 $3y$	0	-2	0	0	0	2

$$\mathbf{F} = \mathbf{K}\mathbf{D} \Leftrightarrow \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{bmatrix} = \frac{k}{5}\begin{bmatrix} 16 & 12 & -16 & -12 & 0 & 0 \\ 12 & 19 & -12 & -9 & 0 & -10 \\ -16 & -12 & 21 & 12 & -5 & 0 \\ -12 & -9 & 12 & 9 & 0 & 0 \\ 0 & 0 & -5 & 0 & 5 & 0 \\ 0 & -10 & 0 & 0 & 0 & 10 \end{bmatrix}\begin{bmatrix} D_{1x} \\ D_{1y} \\ D_{2x} \\ D_{2y} \\ D_{3x} \\ D_{3y} \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} F_{1x} \\ -P \\ 0 \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{bmatrix} = \frac{k}{5}\begin{bmatrix} 16 & 12 & -16 & -12 & 0 & 0 \\ 12 & 19 & -12 & -9 & 0 & -10 \\ -16 & -12 & 21 & 12 & -5 & 0 \\ -12 & -9 & 12 & 9 & 0 & 0 \\ 0 & 0 & -5 & 0 & 5 & 0 \\ 0 & -10 & 0 & 0 & 0 & 10 \end{bmatrix}\begin{bmatrix} 0 \\ D_{1y} \\ D_{2x} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} F_{1x} \\ -P \\ 0 \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{bmatrix} = \frac{k}{5} \begin{bmatrix} 16 & 12 & -16 & -12 & 0 & 0 \\ 12 & 19 & -12 & -9 & 0 & -10 \\ -16 & -12 & 21 & 12 & -5 & 0 \\ -12 & -9 & 12 & 9 & 0 & 0 \\ 0 & 0 & -5 & 0 & 5 & 0 \\ 0 & -10 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 0 \\ D_{1y} \\ D_{2x} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{k}{5} \begin{bmatrix} 19 & -12 \\ -12 & 21 \end{bmatrix} \begin{bmatrix} D_{1y} \\ D_{3x} \end{bmatrix} = \begin{bmatrix} -P \\ 0 \end{bmatrix}$$

Compute the Displacements

$$\Leftrightarrow \mathbf{D_{red}} = \mathbf{K_{red}^{-1}} \mathbf{F_{red}}$$

$$\mathbf{K_{red}^{-1}} = \frac{1}{k} \begin{bmatrix} \frac{7}{17} & \frac{4}{17} \\ \frac{4}{17} & \frac{19}{51} \end{bmatrix}$$

Invert Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\Leftrightarrow \mathbf{D_{red}} = \frac{1}{k} \begin{bmatrix} \frac{7}{17} & \frac{4}{17} \\ \frac{4}{17} & \frac{19}{51} \end{bmatrix} \begin{bmatrix} -P \\ 0 \end{bmatrix} = \frac{1}{k} \begin{bmatrix} \frac{7}{17} \cdot -P + \frac{4}{17} \cdot 0 \\ \frac{4}{17} \cdot -P + \frac{19}{51} \cdot 0 \end{bmatrix} = \begin{bmatrix} \frac{-7P}{17k} \\ \frac{-4P}{17k} \end{bmatrix}$$

$[m] = \frac{[N]}{\frac{[N]}{[m]}} = [m]$ OK

Compute the Forces

$$\Leftrightarrow \begin{bmatrix} F_{1x} \\ -P \\ 0 \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{bmatrix} = \frac{k}{5} \begin{bmatrix} 16 & 12 & -16 & -12 & 0 & 0 \\ 12 & 19 & -12 & -9 & 0 & -10 \\ -16 & -12 & 21 & 12 & -5 & 0 \\ -12 & -9 & 12 & 9 & 0 & 0 \\ 0 & 0 & -5 & 0 & 5 & 0 \\ 0 & -10 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 0 \\ -\frac{7P}{17k} \\ -\frac{4P}{17k} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} F_{1x} \\ -P \\ 0 \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{bmatrix} = P \begin{bmatrix} -\frac{4}{17} \\ -1 \\ 0 \\ \frac{3}{17} \\ \frac{4}{17} \\ \frac{14}{17} \end{bmatrix}$$

$[N] = [N]$ OK

Check!

$$\sum \mathbf{F} = \mathbf{0}$$

$$\Rightarrow \sum \mathbf{F} = P\left(-\frac{4}{17} + (-1) + 0 + \frac{3}{17} + \frac{4}{17} + \frac{14}{17}\right) = 0$$